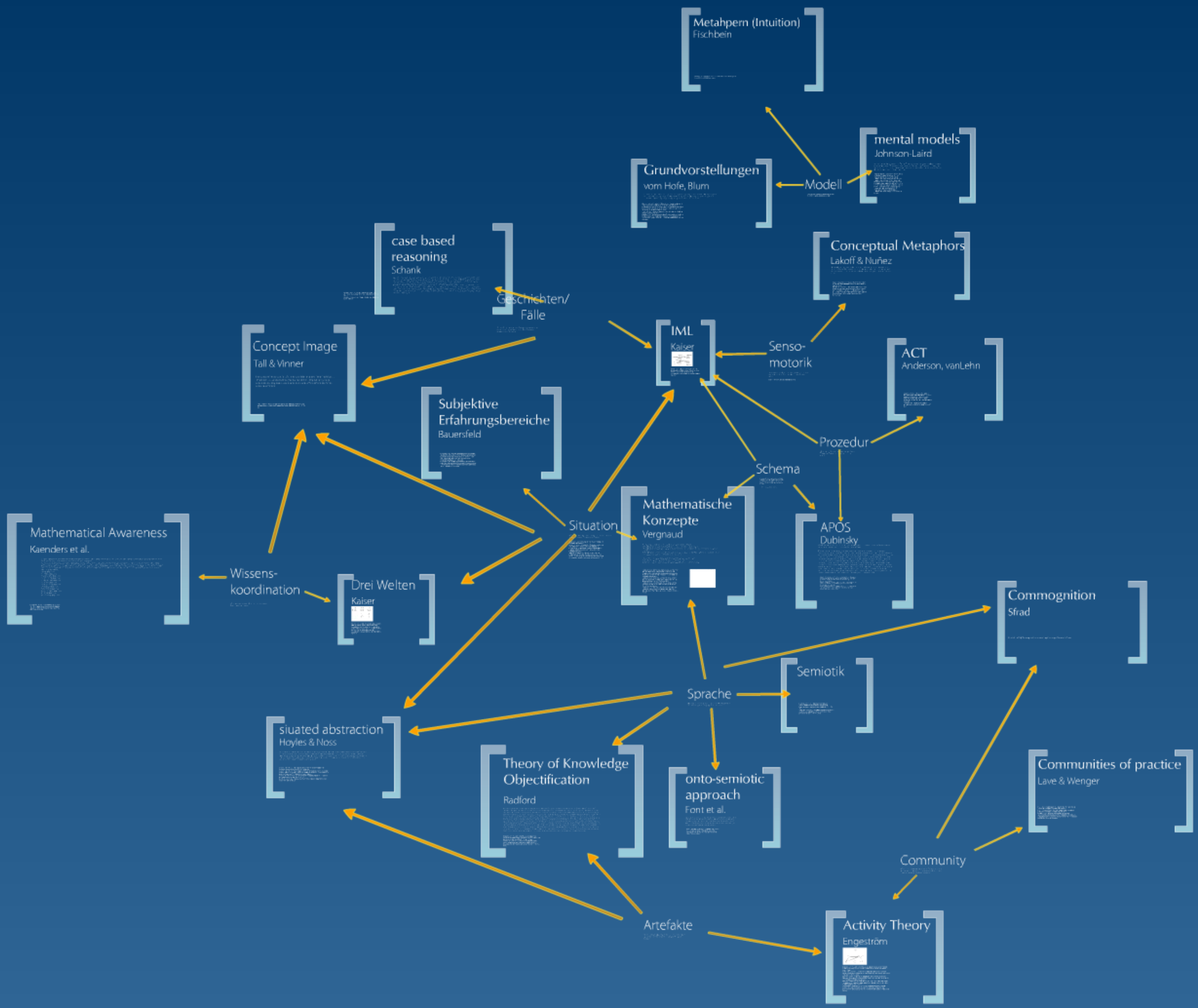

Metahpern (Intuition)
Fischbein
mental models
Johnson-Laird
Grundvorstellungen
vom Hofe, Blum
Modell
Conceptual Metaphors
Lakoff & Nuñez
case based reasoning
Schank
Geschichten/ Fälle
Concept Image
Tall & Vinner
IML
Kaiser
Senso-motorik
ACT
Anderson, vanLehn
Subjektive Erfahrungsbereiche
Bauersfeld
Prozedur
Schema
Mathematische Konzepte
Vergnaud
APOS
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Kaenders et al.
Wissens-koordination
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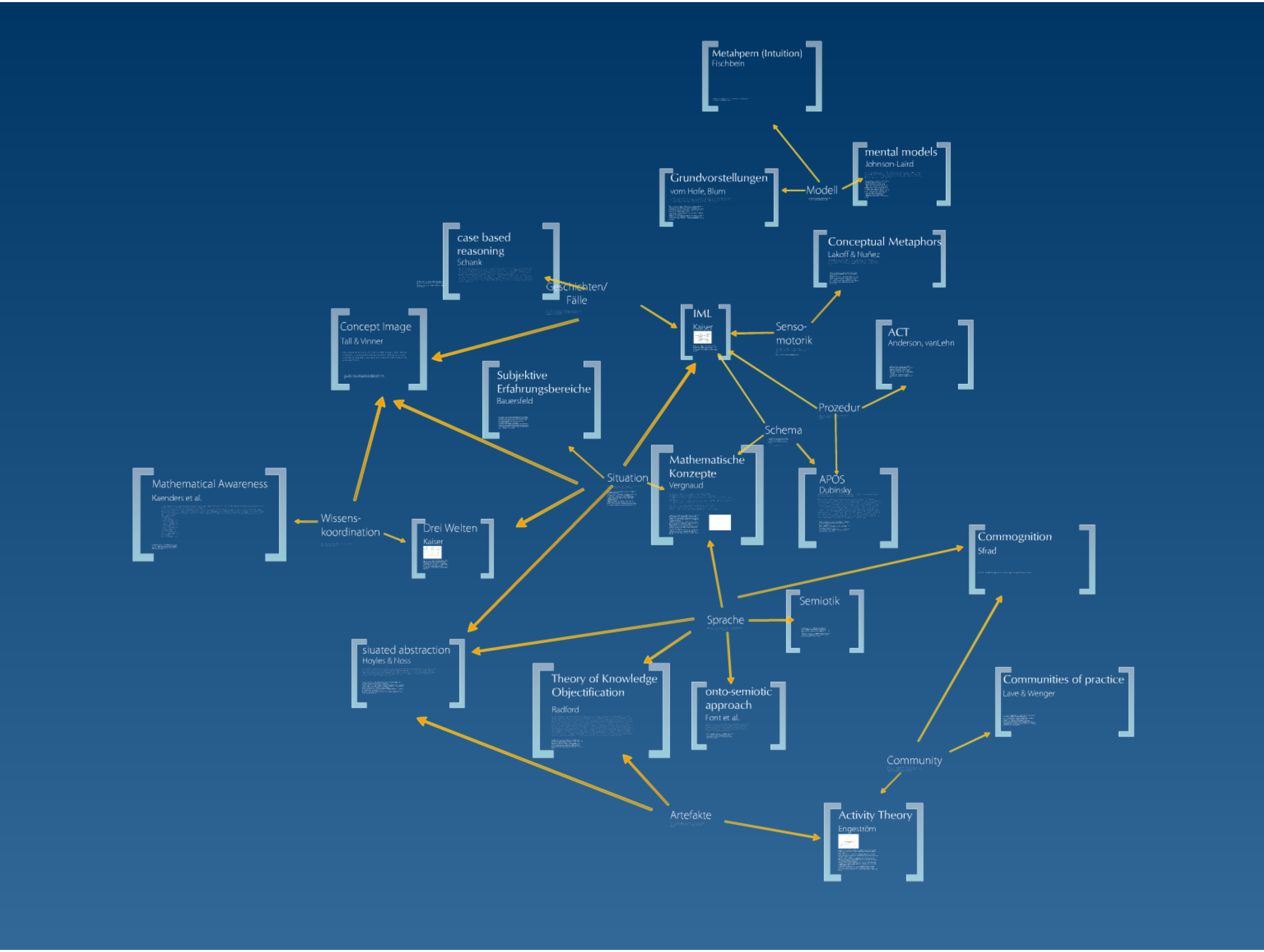

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mental models

Johnson-Laird

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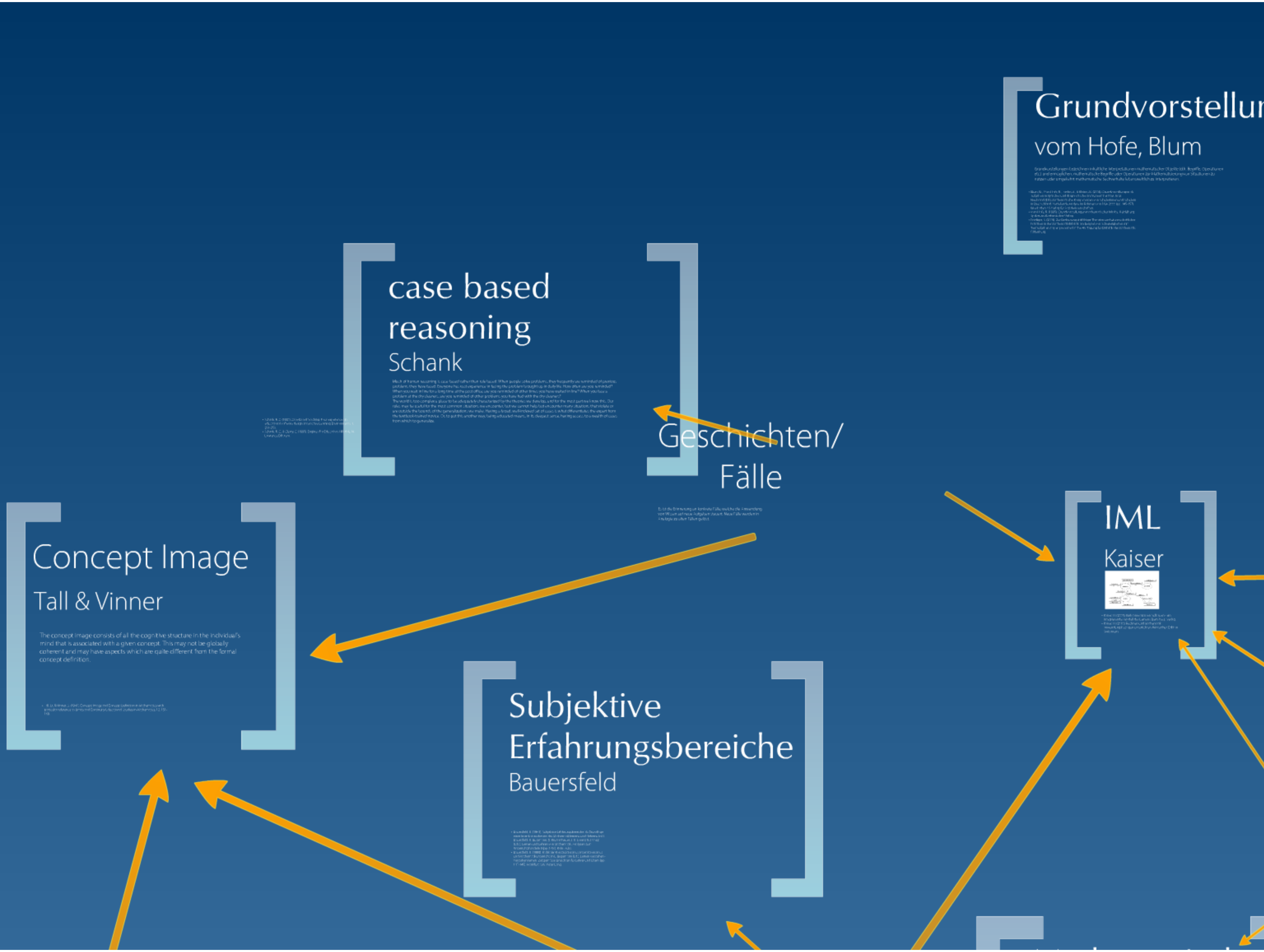
- Johnson-Laird, P. N., & Mark, S. (1978). The Psychology of Syllogisms. Cognitive Psychology, 10, 64-99.
- Johnson-Laird, P. N. (1981). Mental models in cognitive science. In D. A. Norman, Perspectives on cognitive science (Ed.), (pp. 147-192). Norwood N. J
- Fischbein, E. (1989). Tacit models and mathematical reasoning. For the learning of mathematics, 9 (2), 9-14.
- Gott, S. P., Hall, E. P., Pokorny, R. A., Dibble, E., & Glaser, R. (1993). A naturalistic study of transfer: Adaptive expertise in technical domains. In D. K. D. R. J. Sternberg (Ed.), Transfer on trial: Intelligence, cognition, and construction (pp. 258-288). Norwood, NJ: Ablex.



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case based reasoning
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Subjektive Erfahrungsbereiche
Bauersfeld

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Geschichten/ Fälle

Es ist die Erinnerung an konkrete Fälle, welche die Anwendung von Wissen auf neue Aufgaben steuert. Neue Fälle werden in Analogie zu alten Fällen gelöst.

case based reasoning

Schank

Much of human reasoning is case based rather than rule based. When people solve problems, they frequently are reminded of previous problems they have faced. Everyone has vast experience in facing the problem brought up in daily life. How often are you reminded? When you wait in line for a long time at the post office, are you reminded of other times you have waited in line? When you face a problem at the dry cleaners, are you reminded of other problems you have had with the dry cleaners?

The world is too complex a place to be adequately characterized by the theories we develop, and for the most part we know this. Our rules may be useful for the most common situations we encounter, but we cannot help but encounter many situations that violate or are outside the bounds of the generalizations we make. Having a broad, well-indexed set of cases is what differentiates the expert from the textbook-trained novice. Or, to put this another way, being educated means, in its deepest sense, having access to a wealth of cases from which to generalize.

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Concept Image

Tall & Vinner

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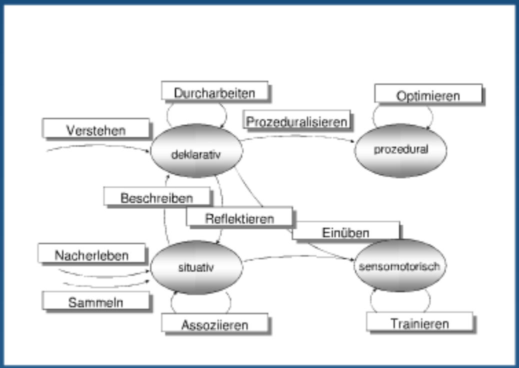

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Conceptual Metaphors

Lakoff & Nuñez

Neurale Mechanismen, welche Wahrnehmung und Bewegungssteuerung gewährleisten, können im "Leerlauf" auch anderen Strukturen Halt geben. Wenn wir uns ein Konzept gleich vorstellen wie wir uns beispielsweise eine Bewegung vorstellen, dann sind wir auf stabilem Grund.

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IML

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Senso-motorik

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ACT

Anderson, vanLehn

- Anderson, J. R. (1989). A theory of the origins of human knowledge. Artificial Intelligence, 40, 313-352.
- Anderson, J. R., Reder, L. M., & Simon, H. A. (1998). Applications and misapplications of cognitive psychology to mathematics education. Unpublished manuscript.
- VanLehn, K. (1990). Mind Bugs: The origins of procedural misconceptions. Cambridge, Mass.: MIT Press.

Prozedur

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Schema

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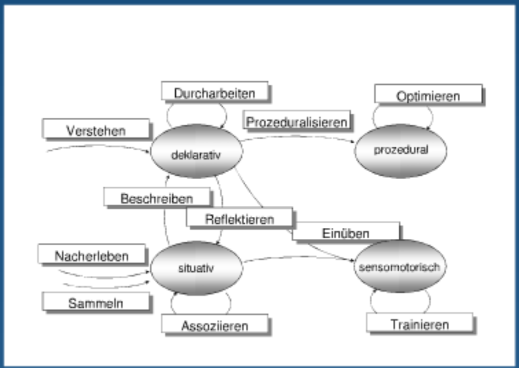

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Schema

Situation

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Vergnaud

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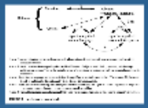

APOS

Dubinsky

APOS Theory adheres to the principle that there is a close relationship between the nature of a mathematical concept and its development in the mind of an individual (Piaget).

The main mechanisms are called interiorization and encapsulation and the related structures are actions, processes, objects, and schemas. The theory postulates that a mathematical concept begins to be formed as one applies a transformation on objects to obtain other objects. A transformation is first conceived as an action, in that it requires specific instruction as well as the ability to perform each step of the transformation explicitly. ... As an individual repeats and reflects on an action, it may be interiorized into a mental process. A process is a mental structure that performs the same operation as the action being interiorized, but wholly in the mind of the individual, thus enabling her or him to imagine performing the transformation without having to execute each step explicitly. If one becomes aware of a process as a totality, realizes that transformations can act on that totality and can actually construct such transformations (explicitly or in one's imagination), then we say the individual has encapsulated the process into a cognitive object. While these structures describe how an individual may construct a single transformation, a mathematical topic often involves many actions, processes, and objects that need to be organized and linked into a coherent framework, which is called a schema.

Schema

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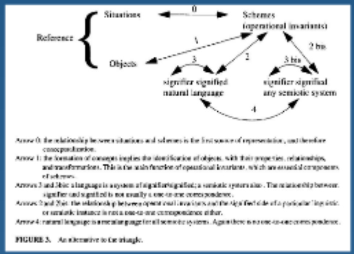

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IML

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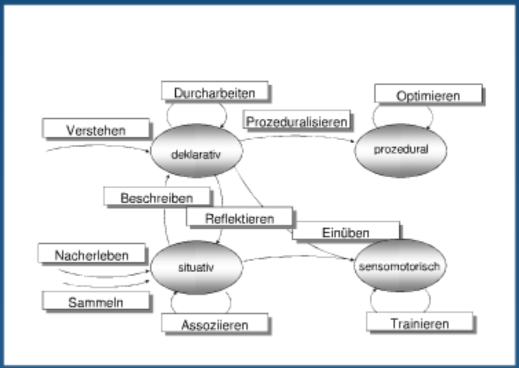

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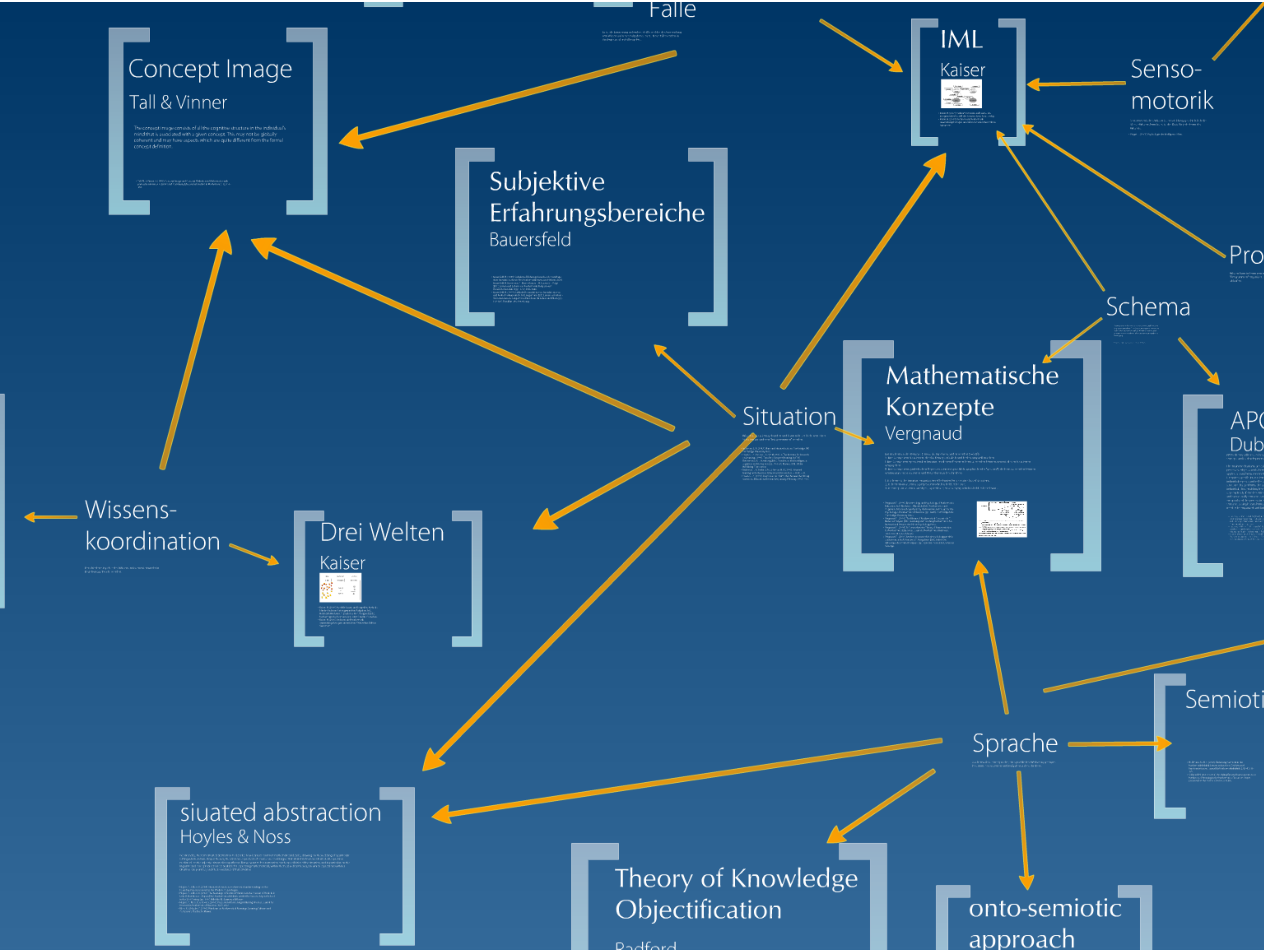

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- Greeno, J. G., Moore, J. L., Smith, D. R., & The Institute for Research on Learning. (1993). Transfer of situated learning. In D. K. Detterman & R. J. Sternberg (Eds.), Transfer on trial: Intelligence, cognition and instruction (pp. 99-167). Norwood, NJ: Ablex Publishing Corporation
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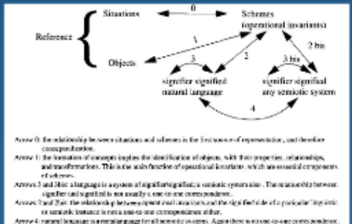


FIGURE 3. An alternative to the triangle.

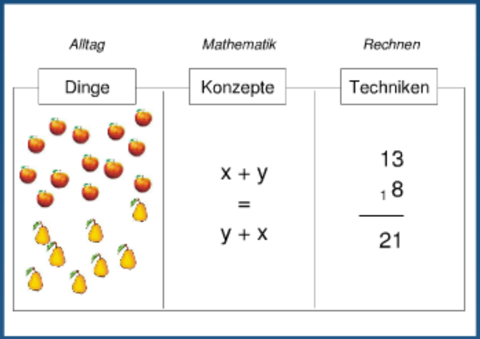
siuated abstraction
Hoyles & Noss

We intend by the term situated abstraction to describe how learners construct mathematical ideas by drawing on the webbing of a particular setting which, in turn, shapes the way the ideas are expressed. Of course, one could argue that all abstractions are situated, all expression mediated - so the adjective situated is superfluous. But we want to focus attention on the specificities of the situation, and in particular, on the linguistic and conceptual resources available for expressing mathematically within them, as well as the ways in which expressions within a situation can point beyond the boundaries of that, situation

- Hoyles, C., & Noss, R. (2004). Situated abstraction: mathematical understandings at the boundary Paper presented at the ICME-10, Copenhagen.
- Hoyles, C., & Noss, R. (2007). The Meanings of Statistical Variation in the Context of Work. In R. Lesh, E. Hamilton & J. Kaput (Eds.), Mathematical Abilities needed for Success beyond School in the 21st Century (pp. 7-35). Hillsdale, NJ: Lawrence Erlbaum
- Hoyles, C., Noss, R., & Pozzi, S. (2001). Proportional Reasoning in Nursing Practice. Journal for Research in Mathematics Education, 32(1), 4-27.
- Noss, R., & Hoyles, C. (1996). Windows on Mathematical Meanings. Learning Cultures and Computers. Dordrecht: Kluwer.

Drei Welten

Kaiser



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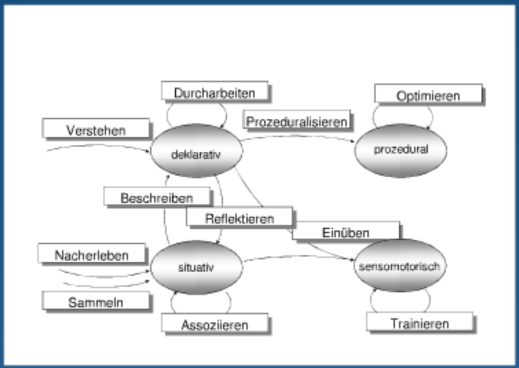
Subjektive Erfahrungsbereiche

Bauersfeld

- Bauersfeld, H. (1983). Subjektive Erfahrungsbereiche als Grundlage einer Interaktionstheorie des Mathematiklernens und -lehrens. In H. Bauersfeld, H. Bussmann, G. Krummheuer, J. H. Lorenz & J. Voigt (Eds.), Lernen und Lehren von Mathematik, Analysen zum Unterrichtshandeln II (pp. 1-56). Köln: Aulis.
- Bauersfeld, H. (1999). Radikaler Konstruktivismus, Interaktionismus und Mathematikunterricht. In E. Begemann (Ed.), Lernen verstehen - Verstehen lernen. Zeitgemässe Einsichten für Lehrer und Eltern (pp. 117-145). Frankfurt a.M.: Peter Lang.

IML

Kaiser



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Concept Image

Tall & Vinner

The concept image consists of all the cognitive structure in the individual's mind that is associated with a given concept. This may not be globally coherent and may have aspects which are quite different from the formal concept definition.

Mathematical Awareness

Kaenders et al.

A mathematical concept consists of contents, of situations, in which mathematical notions can be interpreted, of mathematical objects that give a logical existence to the concept or of problems for whose solution the concept serves as a tool.
For the conceptual framework of mathematical awareness we distinguish three aspects contents, thinking and skills as three main dimensions in which mathematical aptitude can be positioned. The basic idea is the quality of the respective mathematical awareness is not an additional dimension in this diagram but it qualifies the way in which the contents, skills, and thinking are connected, thus it qualifies the way in which for example someone argues in arithmetic by visualizing or someone proves in calculus by algebraizing.

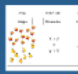
i. Social awareness
ii. Imitative awareness
iii. Manipulative awareness
iv. Instrumental awareness
v. Diagrammatic awareness
vi. Intuitive awareness
vii. Experimental awareness
viii. Strategic awareness
ix. Contextual awareness
x. Argumentative awareness
xi. Logical awareness
xii. Theoretical awareness

Wissens-koordination

Verschiedene Aspekte des Wissens müssen miteinander in Beziehung gebracht werden.

Drei Welten

Kaiser



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Bau

Wissens-koordination

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Mathematical Awareness

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- Kaenders, R. H., Kvasz, L., & Weiss-Pidstrygach, Y. (2011, 9.-13.2.). Recovering Mathematical Awareness by Linguistic Analysis of Variable Substitution. Paper presented at the Creme 7, Rzészov, Polen.

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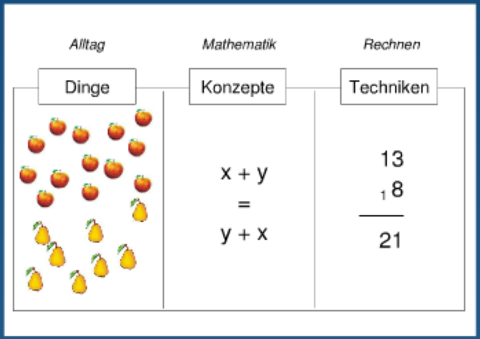

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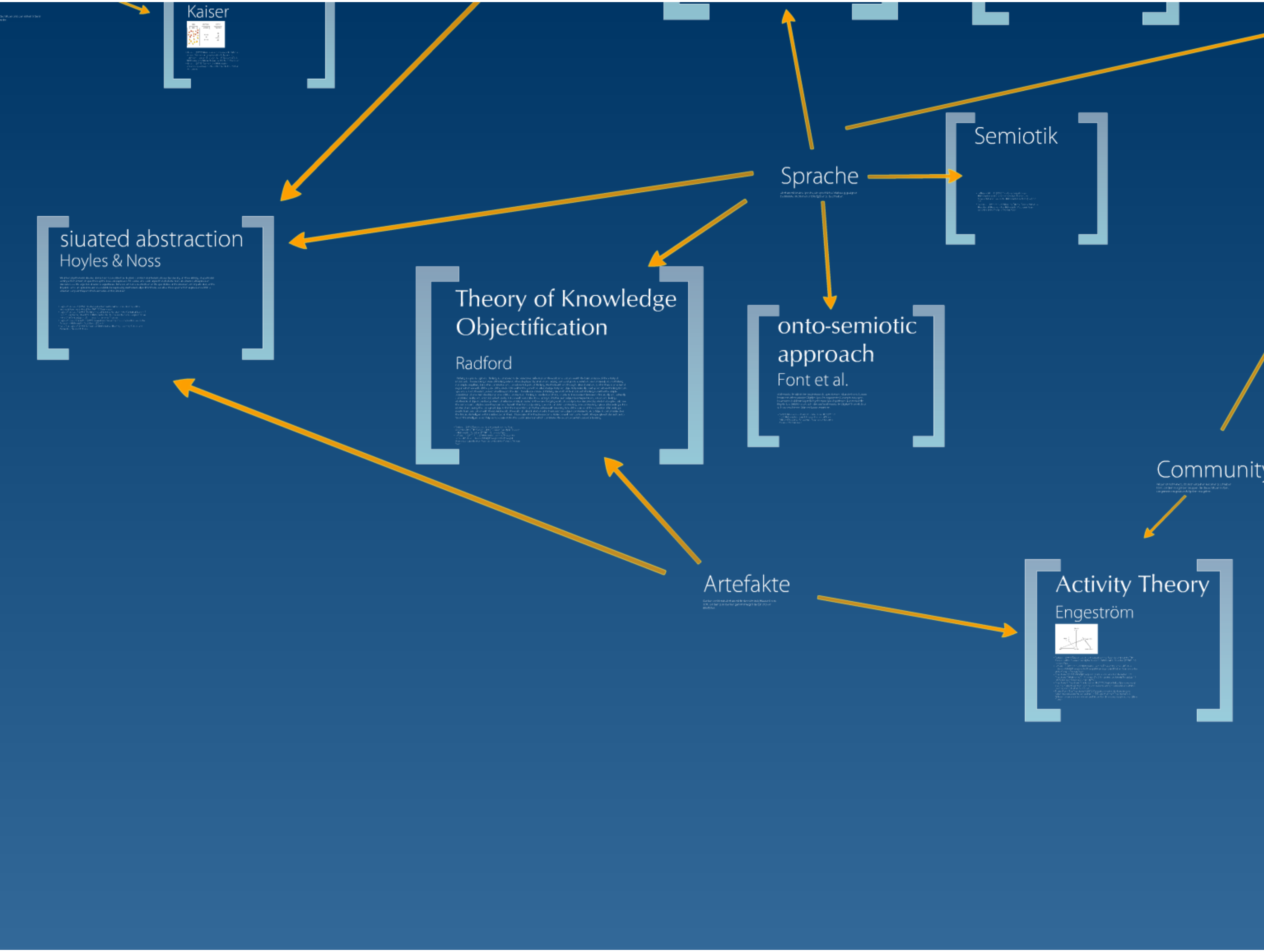
Kaiser
Semiotik
Sprache
siuated abstraction
Hoyles & Noss
Theory of Knowledge Objectification
Radford
onto-semiotic approach
Font et al.
Communit
Artefakte
Activity Theory
Engeström

Artefakte

Denken und damit Mathematik findet nicht im luftleeren Raum statt, sondern zum Denken gehört intergral der Einsatz von Artefakten.

siuated abstraction
Hoyles & Noss

We intend by the term situated abstraction to describe how learners construct mathematical ideas by drawing on the webbing of a particular setting which, in turn, shapes the way the ideas are expressed. Of course, one could argue that all abstractions are situated, all expression mediated - so the adjective situated is superfluous. But we want to focus attention on the specificities of the situation, and in particular, on the linguistic and conceptual resources available for expressing mathematically within them, as well as the ways in which expressions within a situation can point beyond the boundaries of that, situation

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- Noss, R., & Hoyles, C. (1996). Windows on Mathematical Meanings. Learning Cultures and Computers. Dordrecht: Kluwer.

Theory of Knowledge Objectification

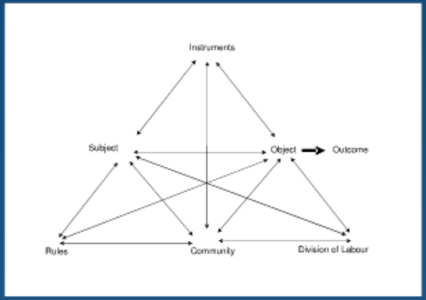
Radford

Thinking is a praxis cogitans. Thinking is considered to be a mediated reflection on the world in accordance with the form or mode of the activity of individuals. The mediating nature of thinking refers to the role played by artefacts in carrying out social practice. Artefacts are not merely aids to thinking nor simple amplifiers, but rather constitutive and consubstantial parts of thinking. We think with and through cultural artefacts. So that there is an external region which we will call the zone of the artefact. It is within this zone that cultural subjectivity and objectivity mutually overlap and where thinking finds its space to act and the mind, extends itself beyond the skin. The reflexive nature of thinking means that the individual's thinking is neither the simple assimilation of an external reality nor an ex nihilo construction. Thinking is a re-flection, that is, a dialectical movement between a historically and culturally constituted reality and an individual who refracts it (as well as modifies it) according to his/her own subjective interpretations, actions and feelings. Mathematical objects are fixed patterns of reflexive activity incrusted in the ever-changing world of social practice mediated by artefacts (People could see the sun as round only because they rounded clay with their hands.). Learning does not consist in constructing or reconstructing a piece of knowledge. It is a matter of endowing the conceptual objects that the student finds in his/her culture with meaning.One of the sources of the acquisition of knowledge results from our contact with the material world, the world of cultural artefacts which surrounds us (objects, instruments, etc.). Objects cannot make clear the historical intelligence that is imbedded in them. This requires that they be used in activities as well as in contact with other people who know how to "read" this intelligence and help us to acquire it. It is this social dimension which constitutes the second essential source for learning.

- Radford, L. (2007). Towards a cultural theory of learning. Paper presented at the Fifth Congress of the European Society for Research in Mathematics Education (CERME – 5), Larnaca, Cyprus.
- LaCroix, L. N. (2011, 9.-13.2.). Mathematics Learning Through the Lenses of Cultural Historical Activity Theory and the Theory of Knowledge Objectification. Paper presented at the Creme 7, Rzészov, Polen.

Activity Theory

Engeström



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- Engeström, Y. (1999). Activity theory and individual and social transformation. In Y. Engeström, R. Miettinen & R. L. Punamaki (Eds.), Perspectives on Activity Theory (pp. 19-38). Cambridge: Cambridge University Press.
- Engeström, Y., Engeström, R., & Karkkainen, M. (1995). Polycontextuality and boundary crossing in expert cognition: Learning and problem solving in complex work activities. Learning and Instruction, 5, 319-336.
- Tuomi-Grohn, T., & Engeström, Y. (2003). Conceptualizing transfer: from standard notions to developmental perspectives. In T. Tuomi-Grohn & Y. Engeström (Eds.), Between school and work: new perspectives on transfer and boundary crossing. Oxford: Elsevier.

Commognition

Sfrad

Sfard, A. (2008) Thinking as Communicating. Cambridge: University Press

Semiotik

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Communities of practice

Lave & Wenger

Community

Activity Theory

Engeström

Community

Wissen ist nicht etwas, das man einzelnen Personen zuschreiben kann, sondern nur ganzen Gruppen, die dieses Wissen nutzen, um gemeinsam gewisse Aufgaben anzugehen.

Commognition

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Sfard, A. (2008) Thinking as Communicating. Cambrige, University Press

Communities of practice

Lave & Wenger

- Lave, J. (1988). Cognition in practice. Mind, mathematics and culture in everyday life. Cambridge: Cambridge University Press.
- Lave, J., & Wenger, E. (1991). Situated Learning. Legitimate peripheral participation. Cambridge: Cambridge University Press.
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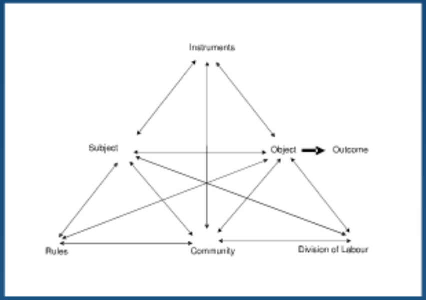
Activity Theory

Engeström



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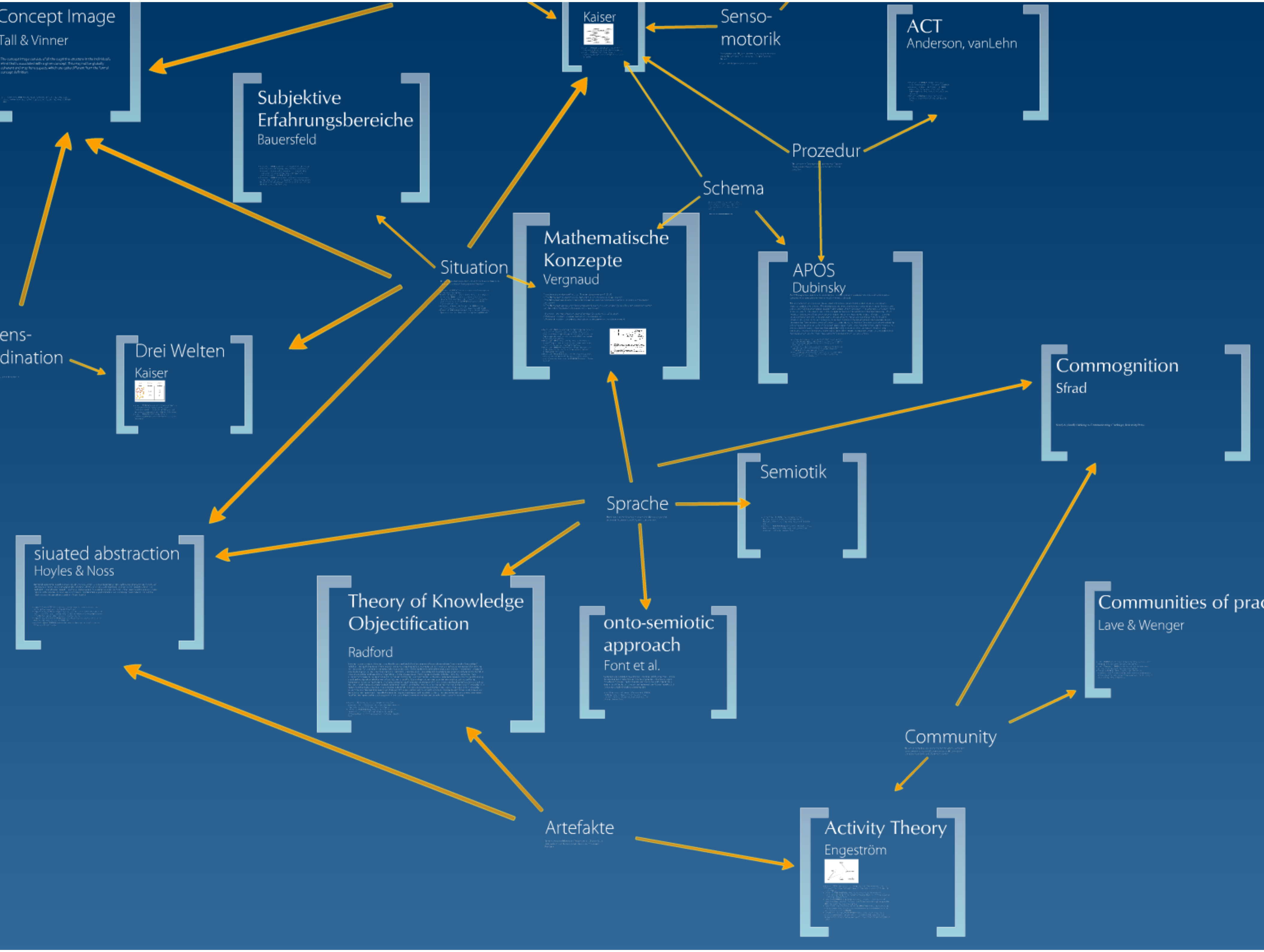
Concept Image
Tall & Vinner
Kaiser
Senso-
motorik
ACT
Anderson, vanLehn
Subjektive
Erfahrungsbereiche
Bauersfeld
Prozedur
Schema
Mathematische
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Vergnaud
Situation
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Drei Welten
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Semiotik
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Font et al.
Communities of prac
Lave & Wenger
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Sprache

Mathematik ist eine Sprache, ein sprachliches Werkzeug, geeignet bestimmte Situationen und Aufgaben zu beschreiben.

Theory of Knowledge Objectification

Radford

Thinking is a praxis cogitans. Thinking is considered to be a mediated reflection on the world in accordance with the form or mode of the activity of individuals. The mediating nature of thinking refers to the role played by artefacts in carrying out social practice. Artefacts are not merely aids to thinking nor simple amplifiers, but rather constitutive and consubstantial parts of thinking. We think with and through cultural artefacts. So that there is an external region which we will call the zone of the artefact. It is within this zone that cultural subjectivity and objectivity mutually overlap and where thinking finds its space to act and the mind, extends itself beyond the skin. The reflexive nature of thinking means that the individual's thinking is neither the simple assimilation of an external reality nor an ex nihilo construction. Thinking is a re-flection, that is, a dialectical movement between a historically and culturally constituted reality and an individual who refracts it (as well as modifies it) according to his/her own subjective interpretations, actions and feelings. Mathematical objects are fixed patterns of reflexive activity incrusted in the ever-changing world of social practice mediated by artefacts (People could see the sun as round only because they rounded clay with their hands.). Learning does not consist in constructing or reconstructing a piece of knowledge. It is a matter of endowing the conceptual objects that the student finds in his/her culture with meaning.One of the sources of the acquisition of knowledge results from our contact with the material world, the world of cultural artefacts which surrounds us (objects, instruments, etc.). Objects cannot make clear the historical intelligence that is imbedded in them. This requires that they be used in activities as well as in contact with other people who know how to "read" this intelligence and help us to acquire it. It is this social dimension which constitutes the second essential source for learning.

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onto-semiotic approach

Font et al.

Mathematische Aktivitäten im Zentrum als operative und dikursive Praxis. Darum herum entstehen primäre Objekte: Sprache, Argumente, Konzepte, Aussagen, Prozeduren, Probleme. Eigentlich geht esum Sprachspiele und konventionelle Regeln. Das Gefühl, dass es sich aber um "mathematische Objekte" handelt, lässt sich aus verschieden Gründen kaum vermeiden.

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Semiotik

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Mathematische Konzepte

Vergnaud

Ein mathematische Konzept C muss als Tripel verstanden werden C=(S,I,R)
S: Eine Menge von Situationen, die das Konzept nützlich und bedeutungsvoll machen
I: Eine Menge von operationalen Invarianten die von Personen benutzt werden können, um mit diesen Situationen umzugehen
R: Eine Menge von symbolischen Representationen (sprachlich, graphisch oder "gestural") die benutzt werden können um Invarianten, Situationen und Prozeduren zu beschreiben.

1. A scheme is the invariant organization of behavior for a certain class of situations.
2. A theorem-in-action is a proposition which is held to be true;
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- Vergnaud, G. (1997). The Nature of Mathematical Concepts. In T. Nunes & P. Bryant (Eds.), Learning and Teaching Mathematics: An International Perspective. Hove: Psychology Press.
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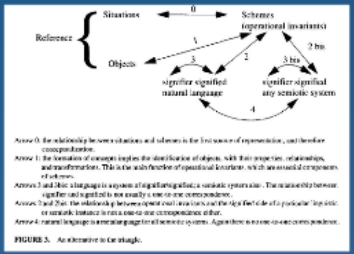

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Senso-motorik

Sensomotorisches Wissen ist entwicklungsgeschichtlich die älteste Wissensform. Es ist die Basis für jede Form des Wissens.

- Piaget, J. (1971). Psychologie der Intelligenz. Olten.

ACT

Anderson, vanLehn

- Anderson, J. R. (1989). A theory of the origins of human knowledge. Artificial Intelligence, 40, 313-352.
- Anderson, J. R., Reder, L. M., & Simon, H. A. (1996). Applications and misapplications of cognitive psychology to mathematics education. Unpublished manuscript.
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Prozedur

Wissen kann in Form von mehr oder weniger flexiblen "Programmen" organisiert sein, die Schritt für Schritt ablaufen.

Schema

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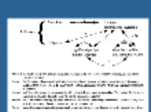

APOS

Dubinsky

APOS Theory adheres to the principle that there is a close relationship between the nature of a mathematical concept and its development in the mind of an individual (Piaget).

The main mechanisms are called interiorization and encapsulation and the related structures are actions, processes, objects, and schemas. The theory postulates that a mathematical concept begins to be formed as one applies a transformation on objects to obtain other objects. A transformation is first conceived as an action, in that it requires specific instruction as well as the ability to perform each step of the transformation explicitly. ... As an individual repeats and reflects on an action, it may be interiorized into a mental process. A process is a mental structure that performs the same operation as the action being interiorized, but wholly in the mind of the individual, thus enabling her or him to imagine performing the transformation without having to execute each step explicitly. If one becomes aware of a process as a totality, realizes that transformations can act on that totality and can actually construct such transformations (explicitly or in one's imagination), then we say the individual has encapsulated the process into a cognitive object. While these structures describe how an individual may construct a single transformation, a mathematical topic often involves many actions, processes, and objects that need to be organized and linked into a coherent framework, which is called a schema.

Co

Sfra

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ACT
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APOS Dubinsky

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- Dubinsky, E., & McDonald, M. (2001). APOS: A Constructivist Theory of Learning in Undergraduate Mathematics Education Research. In D. Holton (Ed.), The teaching and Learning of Mathematics at University Level: An ICMI Study (pp. 273-280): Kluwer Academic Publishers.
- Dubinsky, E., Weller, K., McDonald, M., & Brown, A. (2004). Some historical issues and paradoxes regarding the concept of infinity: An APOS based analysis, Part 1. Educational Studies in Mathematics, 58(3), 335-359.
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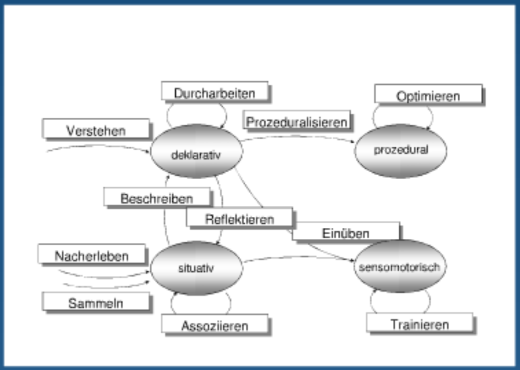
IML

Kaiser



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