

Metaphern (Intuition) Fischbein

Platzhölzer, 1981: Metaphern (Intuition) und Fischbein. In: *Metaphern (Intuition) und Fischbein*. Frankfurt a.M.: Suhrkamp Verlag, 1981.

Grundvorstellungen vom Hofe, Blum

Grundvorstellungen beschreiben intuitive Interpretationen mathematischer Objekte. z.B. Begriffe, Operationen oder Ordnungsbeziehungen. mathematische Begriffe werden Operationen als mathematische Operationen auf Zahlen oder ungelöst mathematische Sachverhalte als mathematische Operationen interpretiert.

→ Hofe, Blum (1981): Grundvorstellungen mathematischer Objekte. In: *Grundvorstellungen mathematischer Objekte*. Frankfurt a.M.: Suhrkamp Verlag, 1981.
→ Hofe, Blum (1981): Grundvorstellungen mathematischer Objekte. In: *Grundvorstellungen mathematischer Objekte*. Frankfurt a.M.: Suhrkamp Verlag, 1981.

Modell

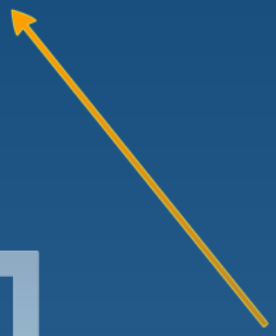
mental models Johnson-Laird

Auch abstrakte Aufgaben werden nicht "buchstäblich" bearbeitet, sondern anhand eines Modells, das dem Problem entspricht. Beispiel: "Aber die Anzahl der Personen ist nicht gleich der Anzahl der Plätze, weil die Plätze nicht alle besetzt sind." (Johnson-Laird, 1983)

→ Johnson-Laird (1983): *Mental Models*. Cambridge: Cambridge University Press, 1983.
→ Johnson-Laird (1983): *Mental Models*. Cambridge: Cambridge University Press, 1983.

Conceptual Metaphors Lakoff & Nuñez

Metaphern sind nicht nur sprachliche Ausdrucksformen, sondern auch kognitive Strukturen, die die Wahrnehmung und das Denken beeinflussen. (Lakoff & Nuñez, 1980)





Modell

Auch abstrakte Aufgaben werden nicht "abstrakt" bearbeitet, sondern anhand eines Modells.

Metaphern (Intuition)

Fischbein

Fischbein, E. (1987) *Intuition in Science and Mathematics. An Educational Approach*.
Dordrecht, D. Reidel Publishing Company.

Grundvorstellungen

vom Hofe, Blum

Grundvorstellungen bezeichnen inhaltliche Interpretationen mathematischer Objekte (d.h. Begriffe, Operationen etc.) und ermöglichen, mathematische Begriffe oder Operationen zur Mathematisierung von Situationen zu nutzen oder umgekehrt mathematische Sachverhalte lebensweltlich zu interpretieren.

- Blum, W., Vom Hofe, R., Jordan, A., & Kleine, M. (2004). Grundvorstellungen als aufgabenanalytisches und diagnostisches Instrument bei PISA. In M. Neubrand (Ed.), *Mathematische Kompetenzen von Schülerinnen und Schülern in Deutschland. Vertiefende Analyse im Rahmen von PISA 2000* (pp. 145-157). Wiesbaden: VS Verlag für Sozialwissenschaften.
- vom Hofe, R. (1995). *Grundvorstellungen mathematischer Inhalte*. Heidelberg: Spektrum Akademischer Verlag.
- Prediger, S. (2009). Zur Bedeutung vielfältiger Theorien und wissenschaftlicher Praktiken in der Mathematikdidaktik am Beispiel von Schwierigkeiten mit Textaufgaben. Paper presented at the 43. Tagung für Didaktik der Mathematik, Oldenburg.

mental models

Johnson-Laird

Auch abstrakte Aufgaben werden nicht "abstrakt" bearbeitet, sondern anhand eines Modells. Jemand, der die Aufgabe hat, aus "Einige Ärzte sind Ausländer, kein Zahnarzt ist Arzt" valide Schlussfolgerungen abzuleiten, stellt sich billich ein paar Ärzte dar und operiert mit diesem Bild - hat in diesem Fall enor Mühe, eine mögliche Schlussfolgerung zu finden.

- Johnson-Laird, P. N., & Mark, S. (1978). The Psychology of Syllogisms. *Cognitive Psychology*, 10, 64-99.
- Johnson-Laird, P. N. (1981). Mental models in cognitive science. In D. A. Norman, *Perspectives on cognitive science* (Ed.), (pp. 147-192). Norwood N. J.
- Fischbein, E. (1989). Tacit models and mathematical reasoning. *For the learning of mathematics*, 9 (2), 9-14.
- Gott, S. P., Hall, E. P., Pokorny, R. A., Dibble, E., & Glaser, R. (1993). A naturalistic study of transfer: Adaptive expertise in technical domains. In D. K. D. R. J. Sternberg (Ed.), *Transfer on trial: Intelligence, cognition, and construction* (pp. 258-288). Norwood, NJ: Ablex.

Grundvorstellung vom Hofe, Blum

Grundvorstellungen können sich als die kognitiven Strukturen darstellen, die die Begriffe, die den Ausdruck von Konzepten bilden, und die die kognitiven Strukturen bilden, die die kognitiven Strukturen bilden, die die kognitiven Strukturen bilden.

Blum, H. (1973). Die kognitiven Strukturen der Grundvorstellungen. In: Blum, H. (Hrsg.), Die kognitiven Strukturen der Grundvorstellungen. Berlin: Springer.

case based reasoning Schank

Case based reasoning is a form of reasoning that is based on the use of past experiences to solve new problems. It is a form of reasoning that is based on the use of past experiences to solve new problems. It is a form of reasoning that is based on the use of past experiences to solve new problems.

• Schank, R. (1982). Case based reasoning. In: Schank, R. (Hrsg.), Case based reasoning. Hillsdale, NJ: Lawrence Erlbaum Associates.

• Schank, R. (1985). Case based reasoning. In: Schank, R. (Hrsg.), Case based reasoning. Hillsdale, NJ: Lawrence Erlbaum Associates.

Geschichten/ Fälle

• Schank, R. (1982). Case based reasoning. In: Schank, R. (Hrsg.), Case based reasoning. Hillsdale, NJ: Lawrence Erlbaum Associates.

Concept Image Tall & Vinner

The concept image consists of all the cognitive structure in the individual's mind that is associated with a given concept. This may not be globally coherent and may have aspects which are quite different from the formal concept definition.

• Tall, D., & Vinner, D. (1981). Concept images and acquisition of mathematical concepts. In: Tall, D., & Vinner, D. (Hrsg.), Concept images and acquisition of mathematical concepts. Hillsdale, NJ: Lawrence Erlbaum Associates.

Subjektive Erfahrungsbereiche Bauersfeld

• Bauersfeld, W. (1984). The ontogenetic development of mathematical thinking in the early childhood years. In: Bauersfeld, W. (Hrsg.), The ontogenetic development of mathematical thinking in the early childhood years. Hillsdale, NJ: Lawrence Erlbaum Associates.

IML Kaiser

• Kaiser, G. (1985). Die kognitiven Strukturen der Grundvorstellungen. In: Kaiser, G. (Hrsg.), Die kognitiven Strukturen der Grundvorstellungen. Berlin: Springer.

• Kaiser, G. (1985). Die kognitiven Strukturen der Grundvorstellungen. In: Kaiser, G. (Hrsg.), Die kognitiven Strukturen der Grundvorstellungen. Berlin: Springer.

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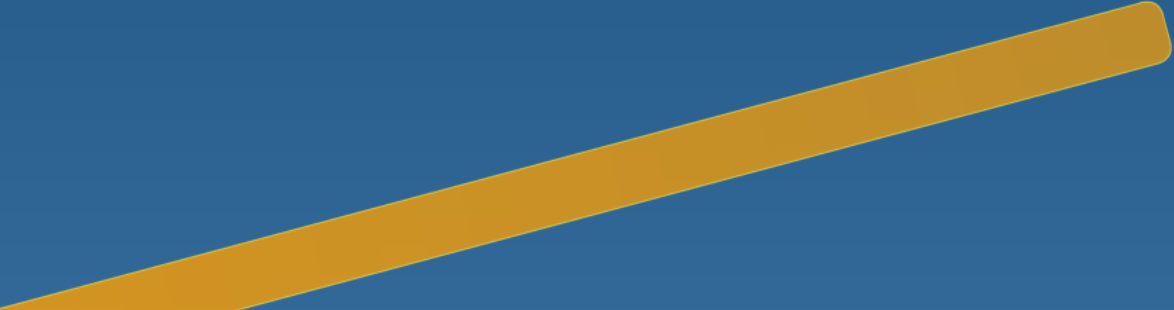
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Geschichten/ Fälle

Es ist die Erinnerung an konkrete Fälle, welche die Anwendung von Wissen auf neue Aufgaben steuert. Neue Fälle werden in Analogie zu alten Fällen gelöst.



case based reasoning Schank

Much of human reasoning is case based rather than rule based. When people solve problems, they frequently are reminded of previous problems they have faced. Everyone has vast experience in facing the problem brought up in daily life. How often are you reminded? When you wait in line for a long time at the post office, are you reminded of other times you have waited in line? When you face a problem at the dry cleaners, are you reminded of other problems you have had with the dry cleaners? The world is too complex a place to be adequately characterized by the theories we develop, and for the most part we know this. Our rules may be useful for the most common situations we encounter, but we cannot help but encounter many situations that violate or are outside the bounds of the generalizations we make. Having a broad, well-indexed set of cases is what differentiates the expert from the textbook-trained novice. Or, to put this another way, being educated means, in its deepest sense, having access to a wealth of cases from which to generalize.



Concept Image

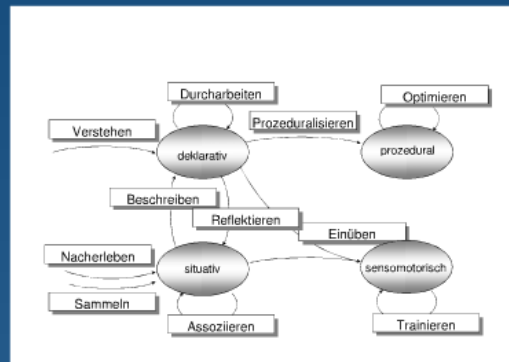
Tall & Vinner

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- Tall, D., & Vinner, S. (1981). Concept Image and Concept Definition in Mathematics with particular reference to Limits and Continuity *Educational Studies in Mathematics*, 12, 151-169.

IML

Kaiser



- Kaiser, H. (2005). Wirksames Wissen aufbauen - ein integrierendes Modell des Lernens. Bern: h.e.p. verlag.
- Kaiser, H. (2010). Rechnen und Mathematik anwendungsbezogen unterrichten. Winterthur: Edition Swissemem.

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Blum, W., vom Hofe, F., Jahnke, S., & Klauß, J. (2004). Grundvorstellungen als aufgabenorientierte, inhaltliche Interpretationen mathematischer Objekte (z.B. Begriffe, Operationen etc.) und ermöglichen, mathematische Begriffe oder Operationen zur Mathematisierung von Situationen zu nutzen oder umgekehrt mathematische Sachverhalte situationswertig zu interpretieren. In: *Mathematische Grundvorstellungen* (Hrsg. von F. Jahnke, S. vom Hofe, W. Blum & J. Klauß), S. 1-17. Münster: Waxmann.

Modell

Auch Modelle, Aufgaben werden nicht "realtät" bezeichnen, sondern sind ein Abbild.

Lakoff, G. & Nuñez, I. (2000). *Where Mathematics Comes From: How the Embodied Mind Brings Mathematics to Life*. New York: Basic Books.

Conceptual Metaphors Lakoff & Nuñez

Neuere Forschungen, welche Wahrnehmung und Bewegungssteuerung gewährleisten, können im "Leben" aufeinander "strukturiert" sein. Wenn wir uns ein Konzept gleich vorstellen wie wir uns beispielsweise eine Bewegung vorstellen, dann sind wir auf einem "Grund".

Lakoff, G., & Nuñez, I. (2000). *Where Mathematics Comes From: How the Embodied Mind Brings Mathematics to Life*. New York: Basic Books.

IML Kaiser



Kaiser, H. (2003). *Integriertes Mathematisches Lernen*. In: *Mathematisches Lernen* (Hrsg. von H. Kaiser), S. 1-17. Münster: Waxmann.

Senso- motorik

Sensomotorisches Wissen ist einstufig sensorisch-motorisch die physische Wissensform. Es ist die Basis für jede Form des Wissens.

Piaget, J. (1971). *Psychologie der Intelligenz*. Göttingen: Vandenhoeck & Ruprecht.

ACT Anderson, vanLehn

Anderson, J. & vanLehn, N. (1978). A Theory of Cognitive Skill Acquisition. *Artificial Intelligence*, 46, 311-342.

Prozedur

Wissensform in Form von mehr oder weniger fixierten "Programmen" organisiert, die Schritt für Schritt ablaufen.

Schema

Senso- motorik

Sensomotorisches Wissen ist entwicklungsgeschichtlich die älteste Wissensform. Es es ist die Basis für jede Form des Wissens.

- Piaget, J. (1971). Psychologie der Intelligenz. Olten.

Conceptual Metaphors

Lakoff & Nuñez

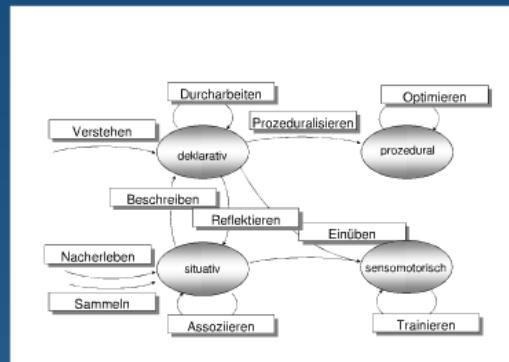
Neurale Mechanismen, welche Wahrnehmung und Bewegungssteuerung gewährleisten, können im "Leerlauf" auch anderen Strukturen Halt geben. Wenn wir uns ein Konzept gleich vorstellen wie wir uns beispielsweise eine Bewegung vorstellen, dann sind wir auf stabilem Grund.

- Lakoff, G., & Núñez, R. (2000). *Where Mathematics Comes From: How the Embodied Mind Brings Mathematics into Being*. New York: Basic Books
- Edwards, L. D. (2011, 9-13.2.). *Embodied Cognitive Science and Mathematics*. Paper presented at the Creme 7, Rzeszow, Polen.
- Font, V., Malaspina, U., Giménez, J., & Wilhelmi, M. R. (2011, 9-13.2.). *Mathematical Objects through the Lens of Three Different Theoretical Perspectives*. Paper presented at the Creme 7, Rzeszow, Polen.



IML

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- Kaiser, H. (2010). Rechnen und Mathematik anwendungsbezogen unterrichten. Winterthur: Edition Swissemem.

IML Kaiser



• Kaiser, H. (2002). *Mathematisches Wissen als Teil einer umfassenderen kulturellen, sozialen, und sprachlichen Welt*.
• Kaiser, H. (2002). *Das Wissen und Lehren über mathematische Beweismethoden*.
• Kaiser, H. (2007). *Prozeduren der Mathematik*.
• Kaiser, H. (2011). *Prozeduren der Mathematik*.
• Kaiser, H. (2011). *Prozeduren der Mathematik*.

Sensomotorik

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• Kaiser, H. (2011). *Prozeduren der Mathematik*.

ACT Anderson, vanLehn

• Anderson, J. R. (1982). *A Theory of Cognitive Skill Acquisition*.
• Anderson, J. R., Leair, J. M., & Smith, H. A. (1986). *Acquisition and Representation of Complex Perceptual-Motor Skills*.
• Anderson, J. R., & Leair, J. M. (1984). *The Structure of Procedural Knowledge*.
• Anderson, J. R., & Leair, J. M. (1984). *The Structure of Procedural Knowledge*.
• Anderson, J. R., & Leair, J. M. (1984). *The Structure of Procedural Knowledge*.

Prozedur

Wissen kann in Form von Regeln oder weniger flexiblen "Programmen" organisiert sein, die Schritt für Schritt ablaufen.

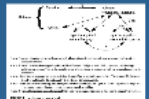
Schema

Ein Schema ist eine mentale Organisation von Wissen, die es ermöglicht, Informationen zu organisieren und zu abrufen.
• Kaiser, H. (2011). *Prozeduren der Mathematik*.
• Kaiser, H. (2011). *Prozeduren der Mathematik*.

Mathematische Konzepte Vergnaud

Entwicklungsorientierte Konzepte (C) müssen als Triplet verstanden werden (C-EU-R).
1. Eine Menge von operationellen Invarianten, die von Personen benutzt werden können, um mit diesen Situationen umzugehen.
2. Eine Menge von mentalen Repräsentationen (sprachlich, graphisch oder "gesteuert"), die benutzt werden können, um Invarianten, Situationen und Prozeduren zu beschreiben.
1. A schema is the invariant organization of behavior for a certain class of situations.
2. A theorem-in-action is a proposition which is held to be true.
3. A concept-in-action is an object, a predicate, or a category which is held to be relevant.

• Vergnaud, G. (1990). *Epistemology and Psychology of Mathematics Education for Research in Mathematics Education and Curriculum Research*.
• Vergnaud, G. (1997). *The Nature of Mathematical Concepts*.
• Vergnaud, G. (2002). *Revisiting the Theory of Action*.
• Vergnaud, G. (2002). *Revisiting the Theory of Action*.
• Vergnaud, G. (2002). *Revisiting the Theory of Action*.



APOS Dubinsky

APOS Theory addresses the principle that there is a close relationship between the nature of a mathematical concept and its development in the mind of an individual (Pogrow).

The main mechanisms are called internalization and encapsulation and the related structures are actions, processes, objects, and schemas. The theory postulates that a mathematical concept begins to be formed as one applies a transformation objects to other objects. A transformation first conceived as an action in that it requires specific instructions as well as the ability to perform each step of the transformation explicitly. As an individual repeats and reflects on an action, it may be internalized into a mental process. A process is a mental structure that performs the same operation as the action being internalized, but wholly in the mind of the individual, thus enabling her/him to imagine performing the transformation without having to execute each step explicitly. For this reason, a process is said to be internalized into a mental process. An individual may actually construct such transformations (explicitly or in one's imagination), then use the individual has encapsulated the process into a cognitive object. While these structures describe how an individual may construct his/her transformation, a mathematical topic often involves many actions, processes, and objects that need to be organized and linked into a coherent framework, which is called a schema.

• Dubinsky, E. (1985). *Internalization and Encapsulation*.
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Situation

Wissen ist situationsgebunden und kann nicht so leicht von einer Situation zur nächsten "mitgenommen" werden.

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• Vergnaud, G. (1997). *The Nature of Mathematical Concepts*.
• Vergnaud, G. (2002). *Revisiting the Theory of Action*.
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Schema

Konzeptionelles Wissen ist in komplexen und flexiblen Strukturen geordnet. Diese Strukturen haben Leerstellen, in die Informationen eingefüllt werden können und bringen so verschiedene Informationen zueinander in Beziehung.

- Piaget, J. (1947) Psychologie der Intelligenz. Rascher

Mathematische Konzepte Vergnaud

Ein mathematische Konzept C muss als Tripel verstanden werden $C=(S,I,R)$

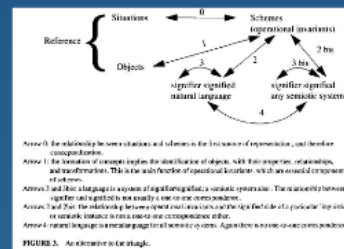
S; Eine Menge von Situationen, die das Konzept nützlich und bedeutungsvoll machen

I: Eine Menge von operationalen Invarianten die von Personen benutzt werden können, um mit diesen Situationen umzugehen

R: Eine Menge von symbolischen Representationen (sprachlich, graphisch oder "gestural") die benutzt werden können um Invarianten, Situationen und Prozeduren zu beschreiben.

1. A scheme is the invariant organization of behavior for a certain class of situations.
2. A theorem-in-action is a proposition which is held to be true;
3. A concept in action is an object, a predicate, or a category which is held to be relevant.

- Vergnaud, G. (1990). Epistemology and Psychology of Mathematics Education. In P. Neshet & J. Kilpatrick (Eds.), Mathematics and Cognition. A Research Synthesis by the International Group for the Psychology of Mathematics Education (pp. 14-80). Cambridge MA: Cambridge University Press.
- Vergnaud, G. (1997). The Nature of Mathematical Concepts. In T. Nunes & P. Bryant (Eds.), Learning and Teaching Mathematics: An International Perspective. Hove: Psychology Press.
- Vergnaud, G. (1998). A Comprehensive Theory of Representation for Mathematics Education. *Journal of Mathematical Behavior*, 17(2), 167-181. (noch lesen)
- Vergnaud, G. (2005). Repères pour une théorie psychologique de la connaissance. In A. Mercier & C. Margolinas (Eds.), Balises en didactique des mathématiques (pp. 123-136). Grenoble: La Pensée Sauvage.





APOS Dubinsky

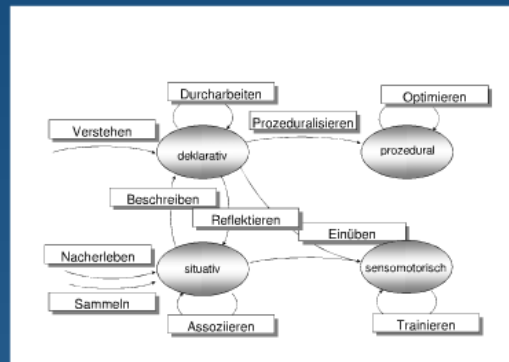
APOS Theory adheres to the principle that there is a close relationship between the nature of a mathematical concept and its development in the mind of an individual (Piaget).

The main mechanisms are called interiorization and encapsulation and the related structures are actions, processes, objects, and schemas. The theory postulates that a mathematical concept begins to be formed as one applies a transformation on objects to obtain other objects. A transformation is first conceived as an action, in that it requires specific instruction as well as the ability to perform each step of the transformation explicitly. ... As an individual repeats and reflects on an action, it may be interiorized into a mental process. A process is a mental structure that performs the same operation as the action being interiorized, but wholly in the mind of the individual, thus enabling her or him to imagine performing the transformation without having to execute each step explicitly. If one becomes aware of a process as a totality, realizes that transformations can act on that totality and can actually construct such transformations (explicitly or in one's imagination), then we say the individual has encapsulated the process into a cognitive object. While these structures describe how an individual may construct a single transformation, a mathematical topic often involves many actions, processes, and objects that need to be organized and linked into a coherent framework, which is called a schema.

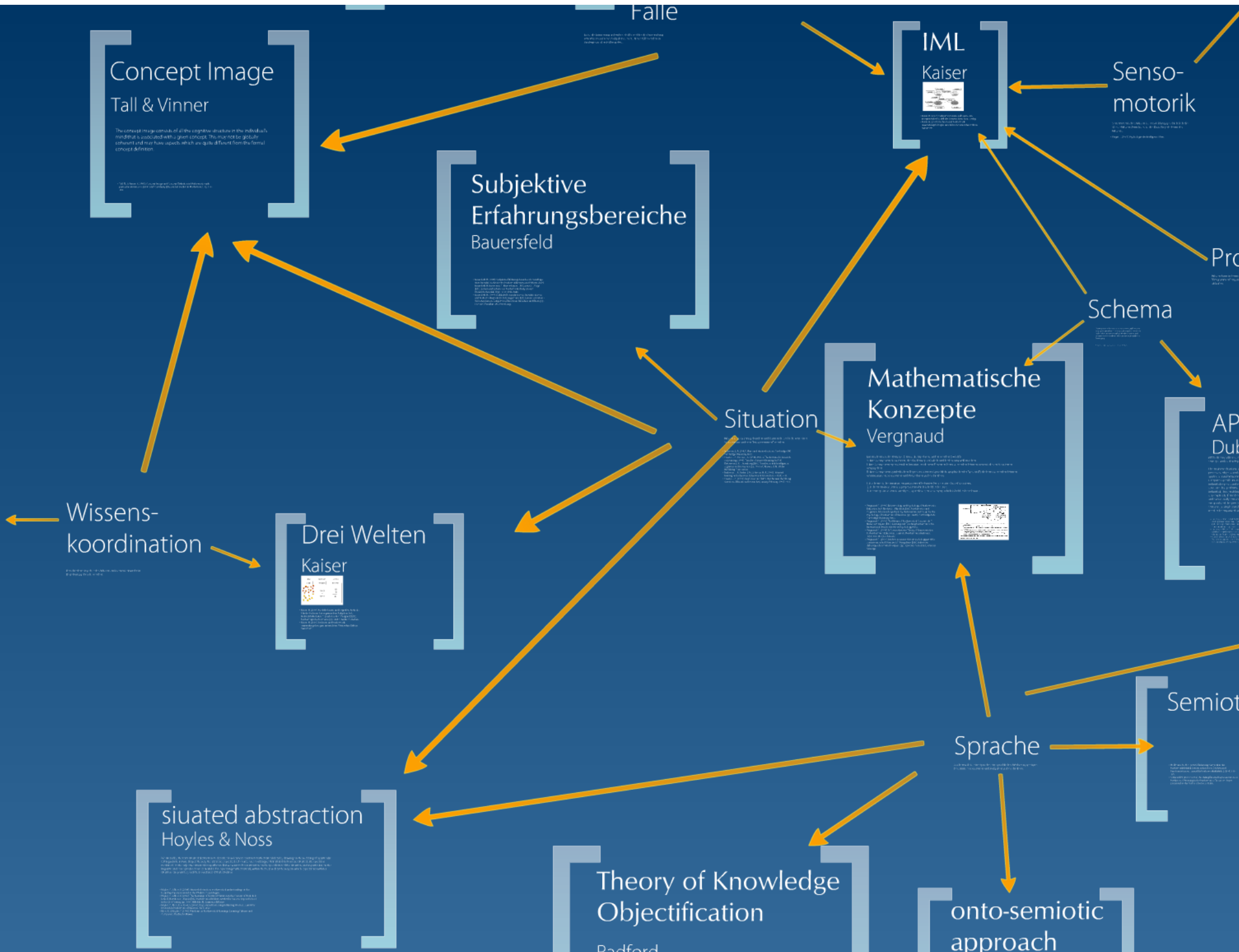
- Dubinsky, E., & McDonald, M. (2001). APOS: A Constructivist Theory of Learning in Undergraduate Mathematics Education Research. In D. Holton (Ed.), *The teaching and Learning of Mathematics at University Level: An ICMI Study* (pp. 273-280): Kluwer Academic Publishers.
- Dubinsky, E., Weller, K., McDonald, M., & Brown, A. (2004). Some historical issues and paradoxes regarding the concept of infinity: An APOS based analysis, Part 1. *Educational Studies in Mathematics*, 58(3), 335-359.
- Font, V., Malaspina, U., Giménez, J., & Wilhelmi, M. R. (2011, 9.-13.2.). *Mathematical Objects through the Lens of Three Different Theoretical Perspectives*. Paper presented at the *Crema 7*, Rzeszów, Polen.

IML

Kaiser



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Situation

Wissen ist situationsgebunden und kann nicht so leicht von einer Situation zur anderen "mitgenommen" werden.

- Suchman, L. A. (1987). Plans and situated actions. Cambridge, UK: Cambridge University Press.
- Greeno, J. G., Moore, J. L., Smith, D. R., & The Institute for Research on Learning. (1993). Transfer of situated learning. In D. K. Detterman & R. J. Sternberg (Eds.), Transfer on trial: Intelligence, cognition and instruction (pp. 99-167). Norwood, NJ: Ablex Publishing Corporation
- Anderson, J. R., Reder, L. M., & Simon, H. A. (1996). Situated learning and education. *Educational Researcher*, 5-11(4), 5-11.
- Greeno, J. G. (1997). Response: On Claims that Answer the Wrong Questions. *Educational Researcher*(January/February 1997), 5-17.

Mathematische Konzepte Vergnaud

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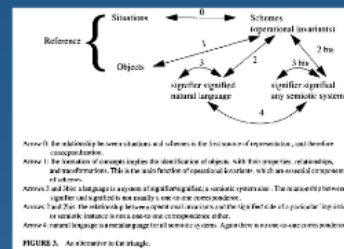
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situated abstraction

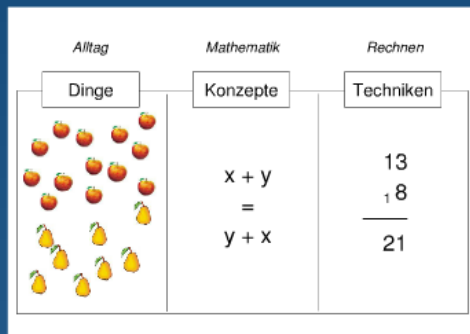
Hoyles & Noss

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- Noss, R., & Hoyles, C. (1996). *Windows on Mathematical Meanings. Learning Cultures and Computers*. Dordrecht: Kluwer.

Drei Welten

Kaiser



- Kaiser, H. (2009). Modelle bauen und begreifen. Mehr als blindes Rechnen bei angewandten Aufgaben. In L. Hefendehl-Hebeker, T. Leuders & H.-G. Weigand (Eds.), *Mathemagische Momente* (pp. 74-85). Berlin: Cornelsen.
- Kaiser, H. (2010). *Rechnen und Mathematik anwendungsbezogen unterrichten*. Winterthur: Edition Swissmem.

Concept Image

Tall & Vinner

The concept image consists of all the cognitive structure in the individual's mind that is associated with a given concept. This may not be globally coherent and may have aspects which are quite different from the formal concept definition.

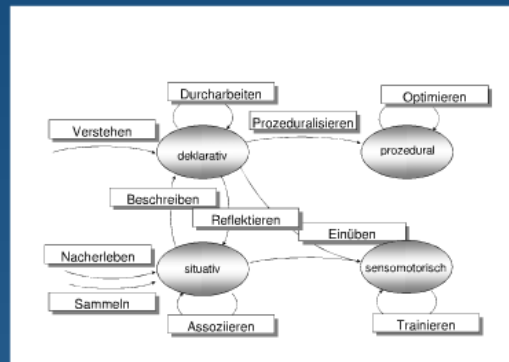
- Tall, D., & Vinner, S. (1981). Concept Image and Concept Definition in Mathematics with particular reference to Limits and Continuity *Educational Studies in Mathematics*, 12, 151-169.

Subjektive Erfahrungsbereiche Bauersfeld

- Bauersfeld, H. (1983). Subjektive Erfahrungsbereiche als Grundlage einer Interaktionstheorie des Mathematiklernens und -lehrens. In H. Bauersfeld, H. Bussmann, G. Krummheuer, J. H. Lorenz & J. Voigt (Eds.), Lernen und Lehren von Mathematik, Analysen zum Unterrichtshandeln II (pp. 1-56). Köln: Aulis.
- Bauersfeld, H. (1999). Radikaler Konstruktivismus, Interaktionismus und Mathematikunterricht. In E. Begemann (Ed.), Lernen verstehen - Verstehen lernen. Zeitgemässe Einsichten für Lehrer und Eltern (pp. 117-145). Frankfurt a.M.: Peter Lang.

IML

Kaiser



- Kaiser, H. (2005). Wirksames Wissen aufbauen - ein integrierendes Modell des Lernens. Bern: h.e.p. verlag.
- Kaiser, H. (2010). Rechnen und Mathematik anwendungsbezogen unterrichten. Winterthur: Edition Swissemem.

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* Tall, D. & T. Vinner, 1981, *Concept Images and Concept Definitions in Mathematics*, *Journal of Mathematical Education in Science and Technology*, 13, 161-166.

Su Er Bau

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www.konrad-zuse.de
Konrad-Zuse-Institut für Informatik
Postfach 10 15 51
D-52074 Aachen
Tel: +49 (0)241 80-2410
Fax: +49 (0)241 80-24110

Mathematical Awareness Kaenders et al.

A mathematical concept consists of contexts, of situations, in which mathematical notions can be interpreted; of mathematical objects that give a logical existence to the concept, and of procedures for whose solution the concept is necessary.

For the conceptual formation of mathematical concepts are different basic aspects: memory, thinking and skills, which should be included in which mathematical methods can be positioned. The focus is on the quality of the aspects: the mathematical awareness is not an additional dimension in the diagram but it qualifies the way in which the concepts, skills, and thinking are connected, that it qualifies the way in which for example someone argues in arithmetic, by visualizing or someone proves a statement by induction.

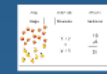
- I. Social awareness
- II. Informative awareness
- III. Algebraic sense awareness
- IV. Theoretical awareness
- V. Organizational awareness
- VI. Intuitive awareness
- VII. Didactical awareness
- VIII. Contextual awareness
- IX. Arguing/Reasoning awareness
- X. Logical awareness
- XI. Theoretical awareness

* Kaenders, F., M. J. L. & P. van Kampen, 2010, *Mathematical Awareness*, *Journal of Mathematical Education in Science and Technology*, 22, 1-10.

Wissens- koordination

Wissenskoordination ist die koordinierende Einwirkung der Basisfunktionen aufeinander.

Drei Welten Kaiser



© Kaiser, G., 1985, *Die drei Welten der Mathematik*, *Journal of Mathematical Education in Science and Technology*, 17, 1-10.



Wissens- koordination

Verschiedene Aspekte des Wissens müssen miteinander in Beziehung gebracht werden.

Mathematical Awareness

Kaenders et al.

A mathematical concept consists of contexts, of situations, in which mathematical notions can be interpreted, of mathematical objects that give a logical existence to the concept or of problems for whose solution the concept serves as a tool.

For the conceptual framework of mathematical awareness we distinguish three aspects contents, thinking and skills as three main dimensions in which mathematical aptitude can be positioned. The basic idea is: the quality of the respective mathematical awareness is not an additional dimension in this diagram but it qualifies the way in which the contents, skills, and thinking are connected, thus it qualifies the way in which for example someone argues in arithmetic by visualizing or someone proves in calculus by algebraizing.

- i. Social awareness
- ii. Imitative awareness
- iii. Manipulative awareness
- iv. Instrumental awareness
- v. Diagrammatic awareness
- vi. Intuitive awareness
- vii. Experimental awareness
- viii. Strategic awareness
- ix. Contextual awareness
- x. Argumentative awareness
- xi. Logical awareness
- xii. Theoretical awareness

- Kaenders, R. H., Kvasz, L., & Weiss-Pidstrygach, Y. (2011, 9-13.2). Recovering Mathematical Awareness by Linguistic Analysis of Variable Substitution. Paper presented at the Creme 7, Rzeszow, Polen.

Concept Image

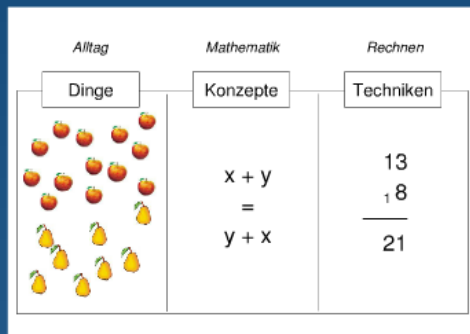
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Drei Welten

Kaiser



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Kaiser



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Semiotik

Sprache

situated abstraction
Hoyles & Noss

Theory of Knowledge
Objectification

Radford

onto-semiotic
approach
Font et al.

Community

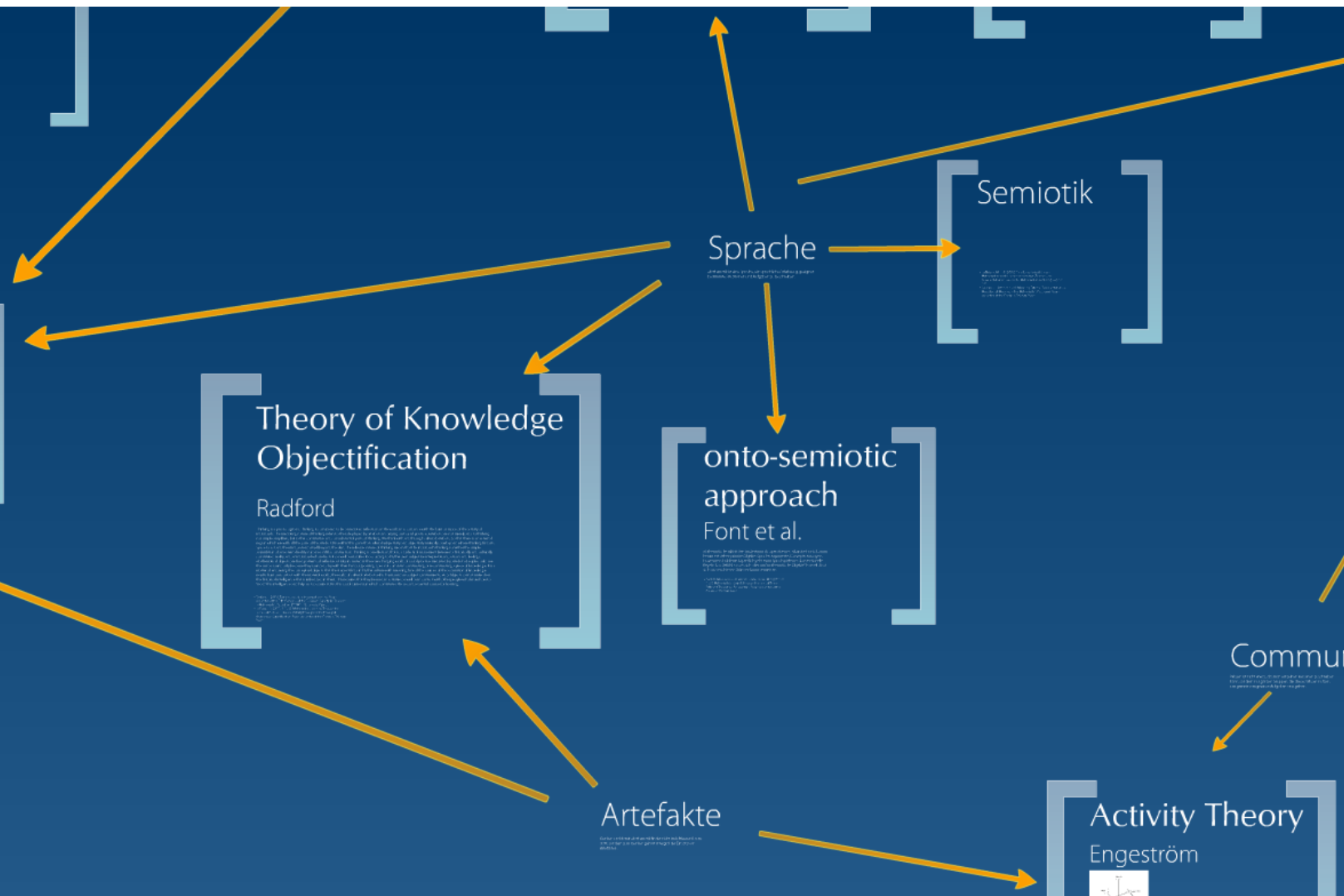
Artefakte

Activity Theory

Engeström



Small text snippet below Engeström.





Artefakte

Denken und damit Mathematik findet nicht im luftleeren Raum statt, sondern zum Denken gehört integral der Einsatz von Artefakten.

situated abstraction

Hoyles & Noss

We intend by the term situated abstraction to describe how learners construct mathematical ideas by drawing on the webbing of a particular setting which, in turn, shapes the way the ideas are expressed. Of course, one could argue that all abstractions are situated, all expression mediated - so the adjective situated is superfluous. But we want to focus attention on the specificities of the situation, and in particular, on the linguistic and conceptual resources available for expressing mathematically within them, as well as the ways in which expressions within a situation can point beyond the boundaries of that, situation

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Theory of Knowledge Objectification

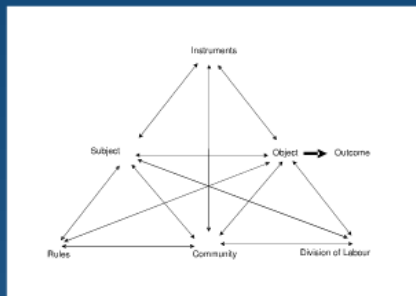
Radford

Thinking is a praxis cogitans. Thinking is considered to be a mediated reflection on the world in accordance with the form or mode of the activity of individuals. The mediating nature of thinking refers to the role played by artefacts in carrying out social practice. Artefacts are not merely aids to thinking nor simple amplifiers, but rather constitutive and consubstantial parts of thinking. We think with and through cultural artefacts. So that there is an external region which we will call the zone of the artefact. It is within this zone that cultural subjectivity and objectivity mutually overlap and where thinking finds its space to act and the mind, extends itself beyond the skin. The reflexive nature of thinking means that the individual's thinking is neither the simple assimilation of an external reality nor an ex nihilo construction. Thinking is a re-flection, that is, a dialectical movement between a historically and culturally constituted reality and an individual who refracts it (as well as modifies it) according to his/her own subjective interpretations, actions and feelings. Mathematical objects are fixed patterns of reflexive activity incrustated in the ever-changing world of social practice mediated by artefacts (People could see the sun as round only because they rounded clay with their hands.). Learning does not consist in constructing or reconstructing a piece of knowledge. It is a matter of endowing the conceptual objects that the student finds in his/her culture with meaning. One of the sources of the acquisition of knowledge results from our contact with the material world, the world of cultural artefacts which surrounds us (objects, instruments, etc.). Objects cannot make clear the historical intelligence that is imbedded in them. This requires that they be used in activities as well as in contact with other people who know how to "read" this intelligence and help us to acquire it. It is this social dimension which constitutes the second essential source for learning.

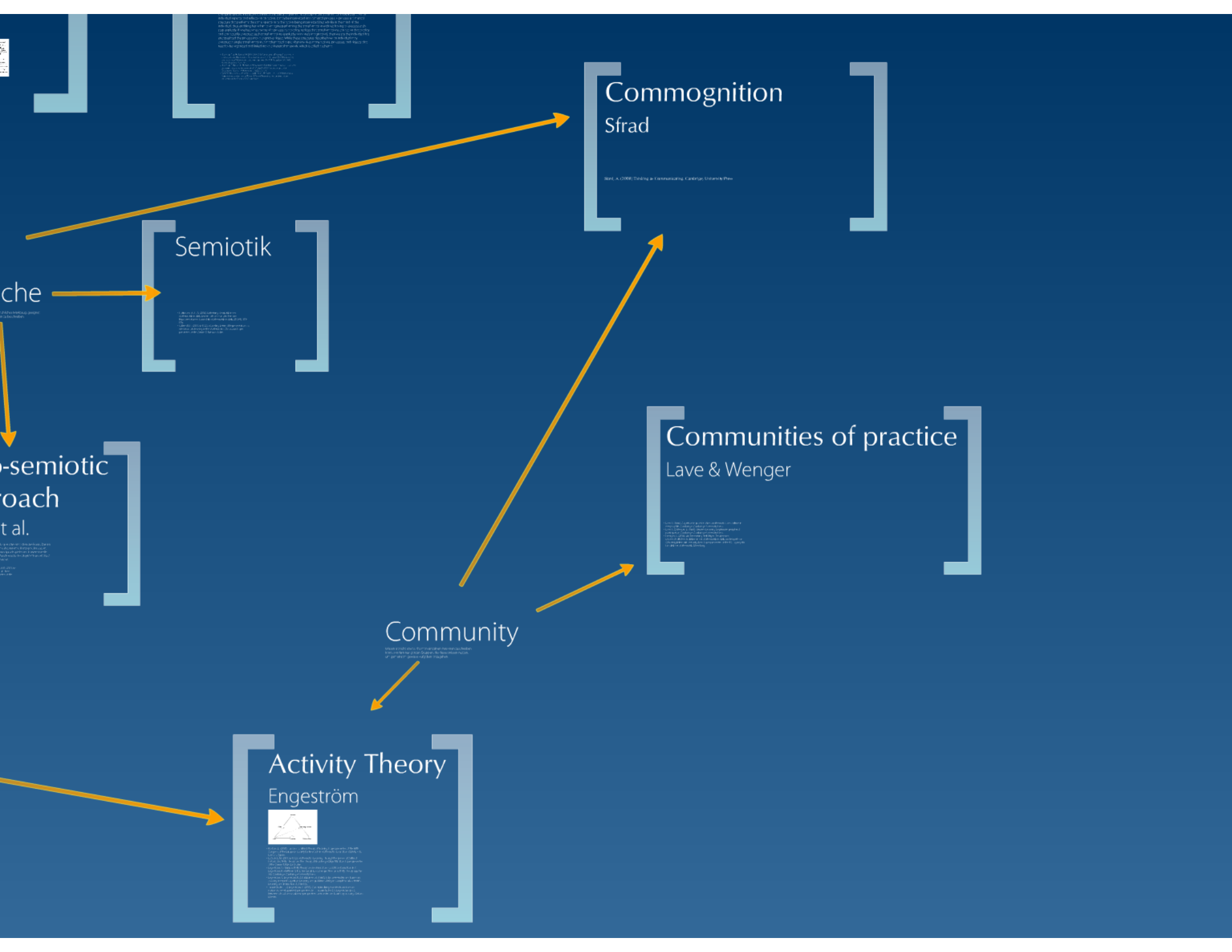
- Radford, L. (2007). Towards a cultural theory of learning. Paper presented at the Fifth Congress of the European Society for Research in Mathematics Education (CERME – 5), Larnaca, Cyprus.
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Activity Theory

Engeström



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- Engeström, Y., Engeström, R., & Karkkainen, M. (1995). Polycontextuality and boundary crossing in expert cognition: Learning and problem solving in complex work activities. *Learning and Instruction*, 5, 319-336.
- Tuomi-Grohn, T., & Engeström, Y. (2003). Conceptualizing transfer: from standard notions to developmental perspectives. In T. Tuomi-Grohn & Y. Engeström (Eds.), *Between school and work: new perspectives on transfer and boundary crossing*. Oxford: Elsevier.



Commognition
Sfrad

Stord, A. (2008) Thinking as Transmutating. Cambridge University Press

Semiotik

che

Post-semiotic
approach
et al.

Communities of practice
Lave & Wenger

Lave & Wenger (1991) Situated Learning: Legitimate Peripheral Participation. Cambridge University Press

Community

Community of Practice (Wenger, 1998)

Activity Theory
Engeström




Engeström (1987) Learning by Expanding and Transforming Activity Systems. In: Engeström, Y. (ed.) Learning by Expanding and Transforming Activity Systems. Cambridge University Press



Community

Wissen ist nicht etwas, das man einzelnen Personen zuschreiben kann, sondern nur ganzen Gruppen, die dieses Wissen nutzen, um gemeinsam gewisse Aufgaben anzugehen.



Commognition

Sfard

Sfard, A. (2008) *Thinking as Communicating*. Cambridge, University Press



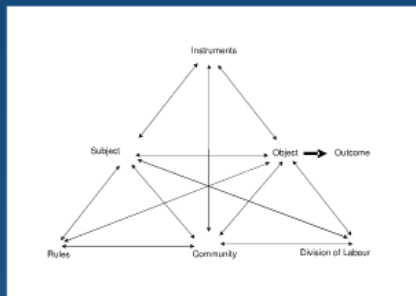
Communities of practice

Lave & Wenger

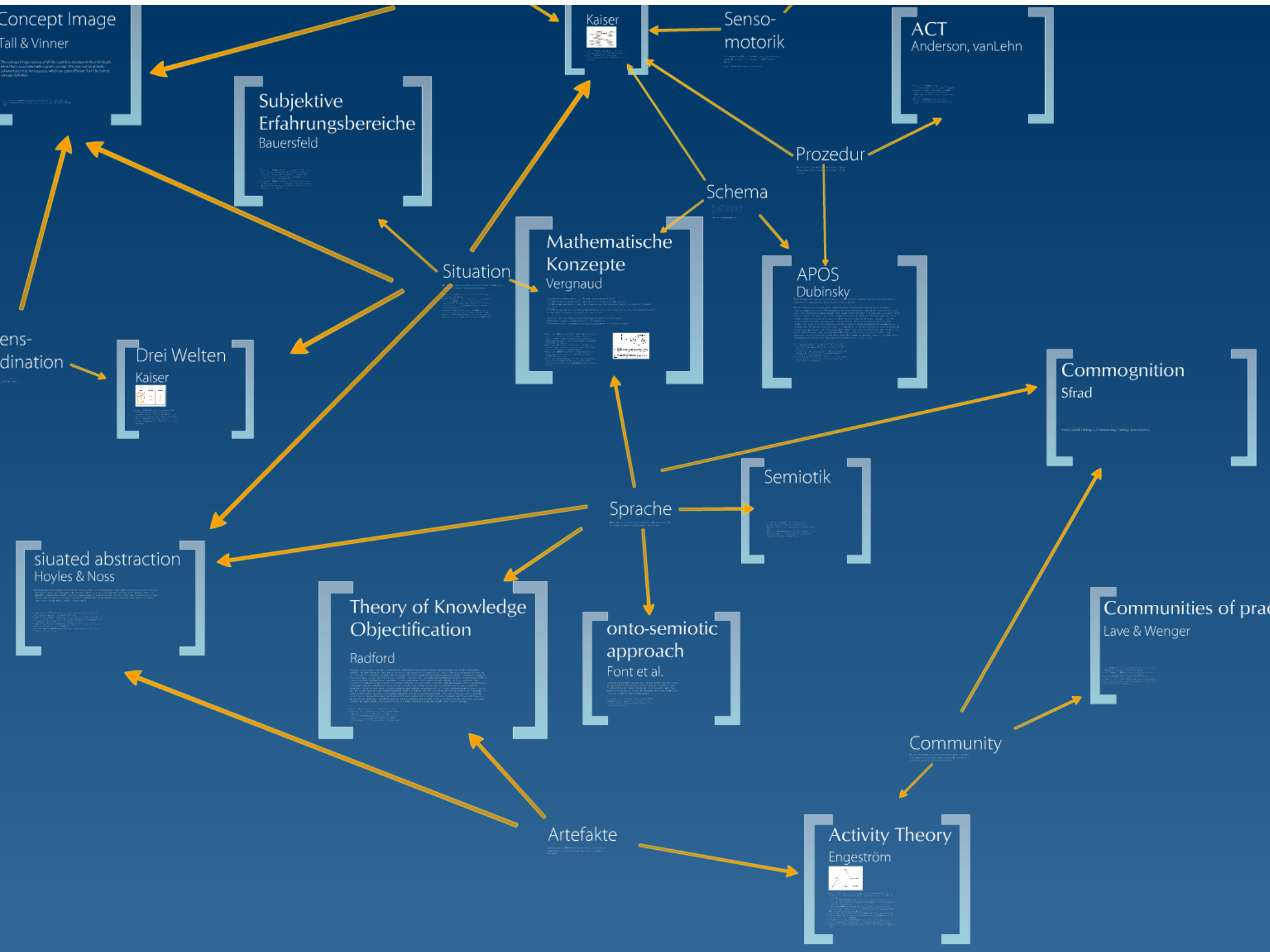
- Lave, J. (1988). *Cognition in practice. Mind, mathematics and culture in everyday life*. Cambridge: Cambridge University Press.
- Lave, J., & Wenger, E. (1991). *Situated Learning. Legitimate peripheral participation*. Cambridge: Cambridge University Press.
- Prediger, S. (2009). Zur Bedeutung vielfältiger Theorien und wissenschaftlicher Praktiken in der Mathematikdidaktik am Beispiel von Schwierigkeiten mit Textaufgaben. Paper presented at the 43. Tagung für Didaktik der Mathematik, Oldenburg.

Activity Theory

Engeström



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Sprache

Mathematik ist eine Sprache, ein sprachliches Werkzeug, geeignet bestimmte Situationen und Aufgaben zu beschreiben.

Theory of Knowledge Objectification

Radford

Thinking is a praxis cogitans. Thinking is considered to be a mediated reflection on the world in accordance with the form or mode of the activity of individuals. The mediating nature of thinking refers to the role played by artefacts in carrying out social practice. Artefacts are not merely aids to thinking nor simple amplifiers, but rather constitutive and consubstantial parts of thinking. We think with and through cultural artefacts. So that there is an external region which we will call the zone of the artefact. It is within this zone that cultural subjectivity and objectivity mutually overlap and where thinking finds its space to act and the mind, extends itself beyond the skin. The reflexive nature of thinking means that the individual's thinking is neither the simple assimilation of an external reality nor an ex nihilo construction. Thinking is a re-flection, that is, a dialectical movement between a historically and culturally constituted reality and an individual who refracts it (as well as modifies it) according to his/her own subjective interpretations, actions and feelings. Mathematical objects are fixed patterns of reflexive activity incrustated in the ever-changing world of social practice mediated by artefacts (People could see the sun as round only because they rounded clay with their hands.). Learning does not consist in constructing or reconstructing a piece of knowledge. It is a matter of endowing the conceptual objects that the student finds in his/her culture with meaning. One of the sources of the acquisition of knowledge results from our contact with the material world, the world of cultural artefacts which surrounds us (objects, instruments, etc.). Objects cannot make clear the historical intelligence that is imbedded in them. This requires that they be used in activities as well as in contact with other people who know how to "read" this intelligence and help us to acquire it. It is this social dimension which constitutes the second essential source for learning.

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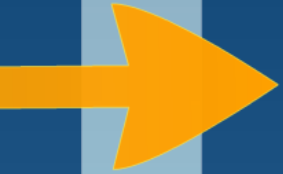
onto-semiotic approach

Font et al.

Mathematische Aktivitäten im Zentrum als operative und diskursive Praxis. Darum herum entstehen primäre Objekte: Sprache, Argumente, Konzepte, Aussagen, Prozeduren, Probleme. Eigentlich geht es um Sprachspiele und konventionelle Regeln. Das Gefühl, dass es sich aber um "mathematische Objekte" handelt, lässt sich aus verschiedenen Gründen kaum vermeiden.

- Font, V., Malaspina, U., Giménez, J., & Wilhelmi, M. R. (2011, 9.-13.2.). Mathematical Objects through the Lens of Three Different Theoretical Perspectives. Paper presented at the Creme 7, Rzeszów, Polen.

Semiotik



- Hoffmann, M. H. G. (2006). Einleitung: Semiotik in der Mathematikdidaktik. Lernen anhand von Zeichen und Repräsentationen. *Journal für Mathematikdidaktik*, 27(3/4), 171-179.
- Sollervall, H. (2011, 9.-13.2.). Modeling External Representations as Mediators of Meaning in the Mathematics Classroom. Paper presented at the Creme 7, Rzeszow, Polen.

Mathematische Konzepte Vergnaud

Ein mathematische Konzept C muss als Tripel verstanden werden $C=(S,I,R)$

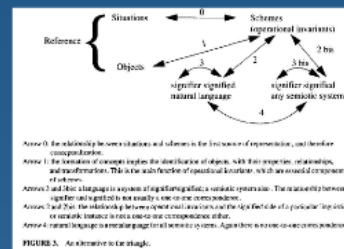
S; Eine Menge von Situationen, die das Konzept nützlich und bedeutungsvoll machen

I: Eine Menge von operationalen Invarianten die von Personen benutzt werden können, um mit diesen Situationen umzugehen

R: Eine Menge von symbolischen Representationen (sprachlich, graphisch oder "gestural") die benutzt werden können um Invarianten, Situationen und Prozeduren zu beschreiben.

1. A scheme is the invariant organization of behavior for a certain class of situations.
2. A theorem-in-action is a proposition which is held to be true;
3. A concept in action is an object, a predicate, or a category which is held to be relevant.

- Vergnaud, G. (1990). Epistemology and Psychology of Mathematics Education. In P. Neshet & J. Kilpatrick (Eds.), Mathematics and Cognition. A Research Synthesis by the International Group for the Psychology of Mathematics Education (pp. 14-80). Cambridge MA: Cambridge University Press.
- Vergnaud, G. (1997). The Nature of Mathematical Concepts. In T. Nunes & P. Bryant (Eds.), Learning and Teaching Mathematics: An International Perspective. Hove: Psychology Press.
- Vergnaud, G. (1998). A Comprehensive Theory of Representation for Mathematics Education. Journal of Mathematical Behavior, 17(2), 167-181. (noch lesen)
- Vergnaud, G. (2005). Repères pour une théorie psychologique de la connaissance. In A. Mercier & C. Margolinas (Eds.), Balises en didactique des mathématiques (pp. 123-136). Grenoble: La Pensée Sauvage.



situated abstraction

Hoyles & Noss

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Commognition

Sfard

Sfard, A. (2008) *Thinking as Communicating*. Cambridge, University Press



IML

Kaiser



• Kaiser H. (2003) Die Kompetenzdimensionen der
Hauptberufliche KIML für Lehrer, Bern: Pöppel Verlag.
• Kaiser H. (2010) Mathematik und Kognitionale
Prozesse in der Mathematikdidaktik. In: W. Blöchl,
H. Kaiser, H. Wittmann (Eds.), *Mathematikdidaktik*,
S. 11-24.

Senso-
motorik

Sensomotorisches Wissen ist eine Lösungsgeschichte die
über die Wissensformen, die es gibt, hinaus für jede Form des
Wissens.

• Piaget, J. (1975) *Psychologie der Intelligenz*. Otfried
Preussner Verlag.

ACT

Anderson, vanLehn

• Anderson, J. R. (1983) *A theory of the control of human information processing*, vol. 2, 112-152.
• Anderson, J. R., Butler, L. A., & Simon, H. A. (1996) *Applications and implications of cognitive psychology to mathematics education* (unpublished manuscript).
• vanLehn, N. (1988) *Intelligence: The origins of procedural representations*. Cambridge, MA: MIT Press.

Prozedur

Wissen kann in Form von mehr oder weniger flexiblen
"Programmen" organisiert sein, die Schritt für Schritt
ablaufen.

Schema

Ein Schema ist ein Modell, das die Struktur und die
Organisation der Informationen darstellt, die in
einer Situation vorliegen. Es ist ein Modell, das die
Organisation der Informationen darstellt, die in
einer Situation vorliegen.

• Craik, F. I. M., & Tulving, E. (1975) *The structure of long-term memory*. In: R. Glaser (Ed.), *Handbook of memory*, S. 1-27. Hillsdale, NJ: Lawrence Erlbaum Associates.

APOS
Dubinsky

APOS Theory adheres to the principle that there is a relationship between the nature of a mathematical
concept and its development in the mind of an individual (Piaget).

The main mechanisms are called internalization and encapsulation and the related structures are actions,
procedures, objects, and schemas. The theory postulates that a mathematical concept begins to be formed as one
applies a transformation on objects to obtain other objects. A transformation is first conceived as an action, in that
it requires specific instructions as well as the ability to perform each step of the transformation explicitly. As an
individual repeats and reflects on an action, it may be internalized into a mental process. A procedure is a mental
structure that performs the same operation as the action being internalized, but without the need to execute each
step explicitly. If one becomes aware of a process as a totality, realizes that transformations can act on that totality
and can actually construct such transformations explicitly (as in algebraic equations), then we say the individual has
encapsulated the procedure into cognitive objects. While these steps proceed in time, an individual may
construct a single transformation, a mathematical topic often involves many actions, procedures, and objects that
need to be organized and linked into a coherent framework, which is called a schema.

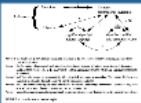
• Dubinsky, E., & Goldfarb, M. (2007). *APOS Theory of Learning*.
Unpublished manuscript, University of Illinois at Chicago, IL, 2007.
• Dubinsky, E. (1985). *The APOS Theory of Learning*. In: *Handbook of Research on Learning and Instruction*, S. 11-27. Hillsdale, NJ: Lawrence Erlbaum Associates.
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Unpublished manuscript, University of Illinois at Chicago, IL, 2007.

Mathematische
Konzepte
Vergnaud

Ein mathematisches Konzept C muss als Teil verstanden werden, sodass:
1. Eine Menge von Situationen, die das Konzept nützlich und bedeutungsvoll machen.
2. Eine Menge von operativen Merkmalen, die von Personen benutzt werden können, um mit diesen Situationen
umzugehen.
3. Eine Menge von mathematischen Repräsentationen (z.B. Text, Graphen, Diagramme), die benutzt werden können
um Situationen, Situationen und Prozesse zu visualisieren.

1. A schema is the invariant organizational behavior for a certain class of situations.
2. A schema is a concept in a presentation which is held to be true.
3. A concept is an object, a special case of a category which is held to be relevant.

• Vergnaud, G. (1990) *Didactic and Psychology of Mathematical
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• Vergnaud, G. (1990) *Didactic and Psychology of Mathematical
Education: The Case of Fractions*. In: *Handbook of Mathematical
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Co
Sfr

Prozedur

Wissen kann in Form von mehr oder weniger flexiblen "Programmen" organisiert sein, die Schritt für Schritt ablaufen.

ACT

Anderson, vanLehn

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APOS Dubinsky

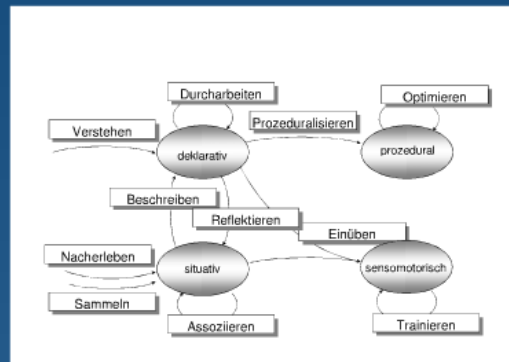
APOS Theory adheres to the principle that there is a close relationship between the nature of a mathematical concept and its development in the mind of an individual (Piaget).

The main mechanisms are called interiorization and encapsulation and the related structures are actions, processes, objects, and schemas. The theory postulates that a mathematical concept begins to be formed as one applies a transformation on objects to obtain other objects. A transformation is first conceived as an action, in that it requires specific instruction as well as the ability to perform each step of the transformation explicitly. ... As an individual repeats and reflects on an action, it may be interiorized into a mental process. A process is a mental structure that performs the same operation as the action being interiorized, but wholly in the mind of the individual, thus enabling her or him to imagine performing the transformation without having to execute each step explicitly. If one becomes aware of a process as a totality, realizes that transformations can act on that totality and can actually construct such transformations (explicitly or in one's imagination), then we say the individual has encapsulated the process into a cognitive object. While these structures describe how an individual may construct a single transformation, a mathematical topic often involves many actions, processes, and objects that need to be organized and linked into a coherent framework, which is called a schema.

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IML

Kaiser



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